Published by Institute of Physics Publishing for SISSA/ISAS

Received: *January 29, 2004*
Revised: *May 31, 2004*
Accepted: *June 3, 2004*

Local string models of soft supersymmetry breaking

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Abstract: We study soft supersymmetry breaking in local models of type II string theory compactifications with branes and fluxes. In such models, magnetic fluxes can be treated as auxiliary fields in $\mathcal{N} = 2$ SUSY multiplets. These multiplets appear as "spurion superfields" in the low-energy effective action for the local model. We discuss the pattern of SUSY breaking from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ to $\mathcal{N} = 0$ in these models, and then identify the fields leading to soft SUSY breaking terms in various examples. In the final section, we reconsider arguments for the Dijkgraaf-Vafa conjecture in gauge theories with softly broken supersymmetry.

Keywords: D-branes, Supersymmetry Breaking, Superstring Vacua.

Contents

1. Introduction

A wide class of string theory backgrounds with low-energy $\mathcal{N} = 1$ supersymmetry in four dimensions is described by combinations of D-branes, orientifold planes, and magnetic fluxes in curved compact manifolds. Gauge dynamics and charged matter will arise from open strings when D-branes wrap cycles and fill the four-dimensional spacetime. Interesting physics may also arise via D-branes which wrap small cycles of the manifold and so give rise to light nonabelian gauge fields and charged matter.

In models consistent with the unification of standard model couplings at a high scale, various directions of the internal manifold are somewhat large compared to the 10-dimensional Planck scale [2]–[6]. The gauge degrees of freedom and the chiral matter will typically be localized. Supersymmetry breaking may occur in some region of the compactification manifold, distinct from the visible sector, perhaps via strong gauge dynamics.

If we wish low-energy SUSY in the visible sector to subdue the hierarchy problem, supersymmetry in the visible sector must be broken by explicit soft terms [7]. In this paper we will focus on the description of tree-level soft SUSY-breaking parameters in local models of D-branes near singularities.[1] In such models, there is a useful softly-broken $\mathcal{N}=2$ structure arising from the underlying $\mathcal{N}=2$ supersymmetric closed-string theory without branes. From this point of view the soft-breaking parameters will be described by auxiliary components of closed string fields, if those fields couple to relevant operators in the brane lagrangian. The auxiliary fields are typically magnetic fluxes with indices along the Calabi-Yau directions. This is the string theory realization of the "spurion" method for describing soft SUSY-violating terms [8].

Such a description of the SUSY-breaking vevs is of interest for a number of reasons.

1. These models describe a local piece of a compactification of some cosmologically interesting compactifications of string theory [9]. The SUSY-breaking can happen elsewhere in the compactification manifold, perhaps via strong gauge dynamics. Such models have led to interesting scenarios such as anomaly mediation [10], gaugino mediation [11], and "tunneling" mediation [12] which use the physical separation in extra dimensions of low-energy degrees of freedom in an essential way. More generally, these constructions allow one to describe physics in a modular fashion — in various throats of the geometry one has a Standard Model module, a supersymmetry breaking module, and possibly an inflation module. These machines communicate only via closed string modes on the CY, which are constrained at string tree level (at least) by the $\mathcal{N}=2$ supersymmetry discussed here.

2. Such a description allows one to plug into the Berkovits formalism [13, 14] for describing the string worldsheet physics for Calabi-Yau compactifications. In this description, the vertex operators come in spacetime supermultiplets, and the expectation values for RR fields do not present an obstruction. This formalism was partial inspiration for this paper, but we will leave an explicit discussion of it for future work.

3. With such a description of SUSY breaking, the underlying supersymmetry is still apparent and one may attempt to make recourse to $\mathcal{N}=1$, $d=4$ SUSY nonrenormalization theorems to compute terms in the effective lagrangian. This has been done in the past in the field theory context, as in [15, 16]. This approach is known to have limitations, particularly if the mass scale set by the soft terms is on the order of or larger than other dynamical scales of the theory. At the end of this paper, we will reconsider the Dijkgraaf-Vafa proposal [17] in the presence of explicit soft SUSY-breaking terms.

Previous work on describing soft SUSY-breaking terms in 4d string models has been

[1] "D-branes near singularities" is meant to be vague: it can refer to space-filling D-branes placed near singularities, or light particle states in four dimensions that arise from Dp-branes wrapping vanishing p-cycles.

done in [18]–[22][2] for a subset of SUSY-breaking terms or for fairly specific models. Soft SUSY-breaking terms in the near-horizon limit are studied in [25]. Soft SUSY-breaking in $\mathcal{N}=2$ gauge theory was studied using the Seiberg-Witten solution in [26]. In this paper we will discuss these terms more generally within the context of type-IIB compactifications, keeping track of the pattern of SUSY breaking via the "spurion" approach, and making contact with the string worldsheet.

The outline of this paper is as follows. In section 2 we will discuss the superspace description of $\mathcal{N}=2$ multiplets for Calabi-Yau moduli in type-II string compactifications, incorporating the discussion of [13], and identifying explicitly the auxiliary fields in terms of fields in $10d$ supergravity. This fleshes out (and modifies some details of) the discussion in [27]. In section 3 we will discuss the explicit breaking of supersymmetry, for gauge dynamics realized either by open strings or by wrapped D-branes. This will include a review of Vafa's derivation [27] of the superpotential for complex stucture moduli [28, 29]. In section 4 we will reconsider the Dijkgraaf-Vafa proposal in the light of explicit soft SUSY-breaking terms. In section 5 we confront our limitations and present conclusions and possibilities for future work.

2. Auxiliary fields for closed string modes

In local type-II models describing D-branes in a noncompact Calabi-Yau background, the closed string fields live in $\mathcal{N}=2$, $d=4$ supermultiplets. Their vevs determine the coupling constants on the worldvolumes of the branes. The underlying $\mathcal{N}=2$ supersymmetry is nonlinearly realized on the D-brane worldvolumes. $\mathcal{N}=2$ supersymmetry is also nonlinearly realized in local models with magnetic fluxes, as Vafa and collaborators have explained [27]–[30]. These two statements are related, as branes can be transmogrified into fluxes by variations of closed string moduli. A particularly striking example of this is the near-horizon limit e.g. [31, 32], as the closed string moduli are driven through topology-changing geometric transitions.

The $\mathcal{N}=2$, $d=4$ transformations of the closed string multiplets constrain the manner in which they appear in the D-brane effective action. For example, "decoupling theorems" in [33, 34] state that to all orders in perturbation theory, closed string hypermultiplets do not couple to the superpotential for open string chiral scalar multiplets.[3] With this in mind, it seems important to understand soft supersymmetry breaking starting with the underlying $\mathcal{N}=2$ structure of closed string degrees of freedom.[4]

[2]While this work was nearing completion, [23, 24] appeared. These papers approach the question of soft supersymmetry breaking on the worldvolumes of D3-branes from a complementary supergravity perspective.

[3]The claim of these references is not that all F-terms for open string degrees of freedom are independent of closed string hypermultiplets. In particular, the gauge coupling for open string vector multiplets is also an F-term. At tree level, the string theory computation is not topological, and for wrapped B(A)-type D-branes in IIB(A), the gauge coupling clearly depends on the volume of the cycle the brane is wrapping, which lies in a hypermultiplet. Note also that at one loop order, the open string gauge coupling is topological [35], but suffers from a holomorphic anomaly [36].

[4]Recent work has been done on theories with nonlinearly realized $\mathcal{N}=2$ supersymmetry in [37]; it would be useful to fit the general discussion in [37] into our framework, but we leave this for future work.

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In this section we will provide a complete identification of $\mathcal{N}=2$ auxiliary fields and closed-string fluxes, using the $\mathcal{N}=2$ superspace description of the massless closed string multiplets provided in [38, 39, 40, 13], by deriving the map between the auxiliary components of the supermultiplets and magnetic fluxes. This superspace formalism is natural from the worldsheet point of view. In either the RNS or the Berkovits-Siegel formalisms, the $\mathcal{N}=2$ supersymmetry algebra is built from a $\mathcal{N}=1$ subalgebra that can be constructed from a left-moving worldsheet current, and a $\mathcal{N}=1$ subalgebra that can be constructed from a right-moving worldsheet current. From the spacetime point of view, one has two copies of $\mathcal{N}=1$ superspace variables, $(\theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ and $(\hat{\theta}^\alpha, \hat{\bar{\theta}}^{\dot{\alpha}})$. In the Berkovits-Siegel formalism, these appear explicitly as anticommuting fields on the worldsheet paired by spacetime supersymmetry to the $4d$ uncompactified target space coordinates. The left-moving supercharges are constructed from the unhatted superspace variables, and the right-moving supercharges are constructed from the superspace variables with hats. The superspace variables form a doublet $(\theta^\alpha, \hat{\theta}^\alpha)$ under the SU(2) R-symmetry.

Although the discussion in [13] takes place within the "hybrid formalism", which is related to the RNS description by a complicated field redefinition [14], the field redefinition is chiral, so that the identification of the superspace coordinates with left- and right-moving supersymmetry currents allows us to understand vertex operators for the auxiliary fields in the RNS formalism, using the techniques of [41, 42]. The worldsheet currents for spacetime supersymmetry contain the spectral flow operators for the internal $c=9$ $\mathcal{N}=(2,2)$ SCFT, together with spin fields for the $4d$ spacetime coordinates. Therefore, if the bottom component of the superfield is an NS-NS field, then one may identify the coefficients of θ^2 or $\hat{\theta}^2$ in the superspace expansion with NS-NS states, and the coefficients of $\theta\hat{\theta}$ with RR states.

We will discuss in turn the superfield description of the vector- and hypermultiplets for type-II compactifications on Calabi-Yau backgrounds. More precisely, we will discuss vector multiplets in type-IIB string theory and hypermultiplets in type-IIA string theory. For vectors in IIA and hypers in IIB, some of the auxiliary fields will be what are loosely called "mirrors of NS flux" [27, 43, 44]. These latter cases will be explored in future work [45]. We will close the section with a brief discussion of the relationship to the hybrid formalism of [14, 13] to our discussion, to address points where our results appear to disagree.

2.1 Massless vector multiplets

In type-IIB string theory compactified on a Calabi-Yau threefold X, the scalar component of the massless vector multiplets are complex structure deformations of the Calabi-Yau background. One may write the full vector multiplet in the $\mathcal{N}=2$ superspace language of [38, 39, 40, 13]. We will discuss here the chiral superfield, which satisfies the constraint [38, 40]:[5]

$$\bar{\nabla}_{\dot{\alpha}} V \equiv \left(-\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\sigma^\mu_{\beta\dot{\alpha}}\theta^\beta \partial_\mu\right) V = 0$$

[5]Here $\sigma^\mu = (1, \vec{\sigma})$ is a four-vector of 2×2 matrices, where $\vec{\sigma}$ are the Pauli matrices and 1 denotes the identity. Furthermore, $\bar{\sigma}^\mu = (1, -\vec{\sigma})$, and $\sigma^{\mu\nu} = \frac{1}{4}\left[\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\right]$.

$$\hat{\bar{\nabla}}_{\dot\alpha} V \equiv \left(-\frac{\partial}{\partial \hat{\bar{\theta}}^{\dot\alpha}} - i\sigma^\mu_{\beta\dot\alpha}\hat\theta^\beta\partial_\mu\right) V = 0\,. \tag{2.1}$$

The superspace expansion for V is:

$$\begin{aligned} V = {} & w^a + \theta^\alpha\zeta^a_\alpha + \hat\theta^\alpha\hat\zeta^a_\alpha + \theta^2 D^a_{++} + \theta^\alpha\hat\theta^\beta\left(\epsilon_{\alpha\beta}D^a_{+-} + F^a_{\alpha\beta}\right) + \\ & + \hat\theta^2 D^a_{--} + \theta^\alpha\hat\theta^2\chi^a_\alpha + \hat\theta^\beta\theta^2\hat\chi^a_\beta + \\ & + \theta^2\hat\theta^2 C^a\,. \end{aligned} \tag{2.2}$$

Here w^a denotes the complex structure deformation. D_{ij} is a symmetric tensor in the SU(2) indices i, j, with complex entries, and is an auxiliary field. C is also a complex auxiliary field. Finally,

$$F_{\mu\nu} = \sigma^{\alpha\beta}_{\mu\nu}F_{\alpha\beta}$$

is an anti-self-dual antisymmetric tensor. This has $16 + 16$ bosonic plus fermionic coordinates.

One may apply further superspace constraints to cut the number of off-shell degrees of freedom in half [38, 39, 40, 13], using the superspace constraint

$$(\epsilon_{ij}\nabla^i\sigma_{\mu\nu}\nabla^j)(\epsilon_{kl}\nabla^k\sigma^{\mu\nu}\nabla^l)V = -96\partial^2\bar V\,. \tag{2.3}$$

Here i is an SU(2) doublet index and $\nabla^i = (\nabla, \hat\nabla)$. In component form the contraints imply:

1. $\partial^2 D_{++} = \partial^2 D^*_{--}$, and $\partial^2 D_{+-}$ is real. Note that this places no constraints on constant modes of D_{ij} [46].

2. $F^a_{\alpha\beta}$ satisfies the identities

$$\sigma^{\alpha\dot\alpha}_\mu\partial_\mu F^a_{\alpha\beta} = \sigma^\mu_{\beta\dot\beta}\partial_\mu F^{a,\dot\alpha\dot\beta} \tag{2.4}$$

   and so is an anti-self-dual abelian vector field strength.

3. $\chi_\alpha = \sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\zeta^{\dot\alpha}$, and $\hat\chi_\alpha = \sigma^\mu_{\alpha\dot\alpha}\partial_\mu\hat{\bar\zeta}^{\dot\alpha}$.

4. $C^a = \partial^2\bar w^a$.

These components comprise one complex scalar, a vector field, an SU(2) triplet of auxiliary fields, and an SU(2) doublet of Weyl fermions. Note that since $\theta, \hat\theta$ correspond respectively to the left- and right-moving supercharges on the worldsheet, D_{+-} is a Ramond-Ramond scalar and $F_{\alpha\beta}$ is a Ramond-Ramond vector, while w and $D_{\pm\pm}$ are NS-NS scalars.

Although this multiplet is clearly simpler, we will work with the less constrained chiral multiplet, letting BRST invariance take care of the reduction to on-shell degrees of freedom. There are at least two reasons that this seems advantageous. First, we will find that the auxiliary fields of the unconstrained multiplet are naturally identified with elements of Dolbeault cohomology in the Calabi-Yau. Secondly, while the chiral constraints (2.1) are linear in derivatives, (2.3) is nonlinear in derivatives. Thus, while the product of two chiral fields is a chiral field, the product of two fields satisfying (2.3) no longer satisfies (2.3).

Auxiliary fields in type IIB on a CY. We wish to begin by stating our results for the identification of auxiliary fields in $\mathcal{N}=2$ vector supermultiplets. In order to state them, it is helpful to review the various descriptions of the moduli space of complex structures, which is the vector multiplet moduli space in type-IIB string theory (and is part of the hypermultiplet moduli space in type-IIA string theory). We follow the discussion in [47, 48, 49, 35]. In the large-volume conformal field theory, the natural description of small deformations of the complex structure is in terms of metric perturbations of the form

$$\delta ds^2 = \delta g_{\bar{i}\bar{j}} d\bar{z}^{\bar{i}} d\bar{z}^{\bar{j}} \,. \tag{2.5}$$

These small deformations modulo reparametrizations correspond to vector-valued holomorphic one-forms

$$\delta g_{\bar{i}\bar{j}} g^{i\bar{i}} d\bar{z}^{\bar{j}} = v^i_{\bar{j}} d\bar{z}^{\bar{j}} \in H^{(0,1)}_{\bar{\partial}}(X, TX) \,. \tag{2.6}$$

This group is isomorphic to the Dolbeault cohomology group $H^{(2,1)}(X)$ by the formula

$$\delta g_{\bar{i}\bar{j}} g^{i\bar{j}} \Omega_{ijk} d\bar{z}^{\bar{i}} dz^j dz^k = \omega_{\bar{i}jk} d\bar{z}^{\bar{i}} dz^j dz^k \,. \tag{2.7}$$

The complex dimension of the moduli space is the complex dimension b^{21} of $H^{(2,1)}(X)$. We will refer to these coordinates as "CFT coordinates".

Another natural set of coordinates on moduli space is the periods of the holomorphic $(3,0)$ form Ω. Let $A^a, B_b, a = 1, \ldots, b^{21}+1$ be a symplectic basis of $H_3(X)$, where

$$A^a \cap B_b = \delta^a_b \,; \qquad B_a \cap A^b = -\delta^b_a \,. \tag{2.8}$$

One may define a dual integral basis α^a, β_b for $H^3(X)$, such that

$$\int_{A^a} \alpha_b = \int_X \alpha_b \wedge \beta^a = \delta^a_b \,; \qquad \int_{B_a} \beta^b = \int_X \alpha_a \wedge \beta^b = -\delta^b_a \,; \qquad \int_{A^a} \beta^b = \int_{B_a} \alpha_b = 0 \,. \tag{2.9}$$

In this basis, the $b^{21}+1$ complex periods

$$t^a = \int_{A^a} \Omega \tag{2.10}$$

form a set of projective coordinates on moduli space. These are projective because the theory is invariant under rescalings

$$\Omega(t) \longrightarrow e^{f(t)} \Omega(t) \tag{2.11}$$

where t are any set of holomorphic coordinates on the moduli space of complex structures. The dual periods

$$F_a(t) = \int_{B_a} \Omega \tag{2.12}$$

are determined by t. Alternatively, we could have picked F as the projective coordinates on moduli space and t as the dual variables. Locally in moduli space, we can write

$$F = \frac{1}{2} \sum_a t^a F_a(t) \,; \qquad F_a = \frac{\partial}{\partial t^a} F \tag{2.13}$$

F is the prepotential for the vector multiplets, and has projective weight two.

If we specify an element of $H^{(2,1)}$ by

$$\omega = \sum_{m=1}^{b_{21}} t^m \omega_m \tag{2.14}$$

where ω_m are basis forms for $H^{(2,1)}(X)$, t^m, $m = (1, \ldots, b_{21})$ are the coordinates in the coordinate system specifying an element of $H^{(2,1)}$, we can write $\Omega = \Omega(t^m)$, and

$$\frac{\partial}{\partial t^m}\Omega = k_m \Omega + \omega_m \tag{2.15}$$

k_m can be shifted by a projective transformation. We can thereby choose a gauge where it vanishes [49].

Using these coordinates we can now describe the auxiliary fields. Two ingredients are the NS-NS three-form $H_{ij\bar{k}}$, and the RR three-form $F_{ij\bar{k}}$. The third ingredient is built from the almost complex structure. For $\mathcal{N} = 2$ Calabi-Yau vacua of type-II string theory, the complex structure J_μ^ν can be written as a two-form by lowering the vector index with the metric:

$$J = J_{\mu\nu} dx^\mu dx^\nu = J_\mu^\lambda g_{\lambda\nu} dx^\mu dx^\nu \,, \tag{2.16}$$

which can be rewritten in complex coordinates:

$$J = J_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} \equiv i g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} \,. \tag{2.17}$$

For $\mathcal{N} = 2$ vacua, $dJ = \partial J = \bar{\partial} J = 0$. We will find that NS auxiliary fields are related to nonvanishing

$$T = i(\partial - \bar{\partial})J \,. \tag{2.18}$$

In the "CFT coordinates", we can expand

$$\begin{aligned}
H &= \sum_m^{b^{21}} h^m \omega_m + H^{(3,0)} + \text{h.c.} \equiv \tilde{H} + H^{(3,0)} + \text{h.c.} \\
F &= \sum_m f^m \omega_m + F^{(3,0)} + \text{h.c.} \equiv \tilde{F} + F^{(3,0)} + \text{h.c.} \\
T &= \sum_m \tau^m \omega_m + T^{(3,0)} + \text{h.c.} \equiv \tilde{T} + T^{(3,0)} + \text{h.c.}
\end{aligned} \tag{2.19}$$

Here we have defined $\tilde{G}$ to indicate the $(2,1)$ part of the three-form G. We will identify

$$\begin{aligned}
D^m_{++} &= (\tau^m + h^m) \\
D^m_{+-} &= g_s(f^m - C^{(0)} h^m) \\
D^m_{--} &= (\tau^m - h^m) \,.
\end{aligned} \tag{2.20}$$

where $C^{(0)}$ is the IIB RR axion. In particular, the auxiliary fields in the chiral vector multiplets are $(2,1)$ forms made from the fluxes and torsion form. In the remainder of this section we will use worldsheet techniques to justify this claim.

Note that this identification does not include the $(3,0)$ piece of H, F, T. In a compact IIB model, the relation

$$i \int_X \Omega \wedge \bar{\Omega} = \frac{4}{3} \int J \wedge J \wedge J \tag{2.21}$$

implies that the deformation of Ω proportional to itself changes the volume of the CY, and hence lies in a hypermultiplet. In such a compact model, this deformation is related by spacetime supersymmetry to the flux along this direction, and hence such flux is the auxiliary field in a hypermultiplet. In a noncompact model, the both hand side of (2.21) is infinite, and the volume deformation does not exist.

Vertex operators for auxiliary fields. The vertex operators for the auxiliary fields can be derived using the techniques in [41, 42], as we shall do here. Since we do not know how to include a nonzero vev for the RR scalar $C^{(0)}$ in the RNS formalism, we will set $C^{(0)} = 0$ in these discussions. We will eventually find the correct dependence of the auxiliary fields on $C^{(0)}$ via spacetime arguments. To begin, let us consider the fields w^a, ζ^a, D^a_{++}, which form a chiral multiplet under the left-moving supersymmetry. In spacetime, the supersymmetry transformations of this chiral multiplet are:

$$\begin{aligned}
[Q_\alpha, w^m] &= \zeta^m_\alpha \\
[Q_{\dot\alpha}, w^m] &= 0 \\
\{Q_\alpha, \zeta^m_\beta\} &= \epsilon_{\alpha\beta} D^m_{++} \\
\{Q_{\dot\alpha}, \zeta^m_\beta\} &= i\partial\!\!\!/_{\dot\alpha\beta} w^m \\
[Q_\alpha, D^m_{++}] &= 0 \\
[Q_{\dot\alpha}, D^m_{++}] &= i\partial\!\!\!/_{\dot\alpha\beta} \zeta^{\beta,m} \,.
\end{aligned} \tag{2.22}$$

Since worldsheet correlators lead to spacetime S-matrix elements, the worldsheet vertex operators are the spacetime fields times the inverse propagator. The result is that acting on the vertex operator for D_{++} twice with the left-moving supersymmetry charge, one gets the vertex operator for the scalar w without any momentum factors. One may use this to guess the vertex operator for D_{++}. Following [41, 42], the vertex operator for w in the $(-1, 0)$ picture is:[6]

$$V_w^{(-1,0)} = e^{-\phi} V_{w,CY} \sim e^{-\phi} \delta g_{\bar{i}\bar{j}} \bar{\psi}^{\bar{i}} \hat{\partial} \bar{X}^{\bar{j}} \,, \tag{2.23}$$

where the final expression is the approximate expression at large radius and complex structure. Here left-moving fermions and derivatives are denoted by unhatted variables, and right-moving fermions and derivatives are denoted by hatted variables. A physical metric variation δg may be expressed in terms of a *harmonic* $(2,1)$ form $\omega_{\bar{i}jk} = \delta g_{\bar{i}\bar{j}} g^{l\bar{j}} \Omega_{ljk}$.

The vertex operator for D_{++} can be written in the $(0,0)$ picture as:

$$V_{D_{++}}(z') = \lim_{z \to z'} \left\{ (z - w) \epsilon^+(z) V_{w,CY}(z') \right\} + \cdots \,. \tag{2.24}$$

The additional pieces are terms in the vertex operator which have nonsingular OPEs with the supercurrents. These will be necessary for V_D to be physical. ϵ^+ is the operator

[6] Cf. ref. [50] for discussions of pictures and of the superconformal ghosts.

generating one unit of spectral flow from the NS sector back to the NS sector, and at large volume and complex structure it can be written [41]:

$$\epsilon^+(z) \sim \Omega_{ijk}\psi^i\psi^j\psi^k \tag{2.25}$$

where Ω is the holomorphic $(3,0)$ form defining the complex structure of X.

Thus, in the $(0,0)$ picture, at large volume and complex structure, we can write:

$$V_{D_{++}} = \delta g_{\bar{i}\bar{j}} g^{i\bar{j}} \Omega_{ijk}\psi^j\psi^k \hat{\partial}\bar{X}^{\bar{i}} = \omega_{\bar{i}jk}\psi^j\psi^k\hat{\partial}\bar{X}^{\bar{i}} + \cdots ; \tag{2.26}$$

a label $m = 1 \ldots b^{21}$ on V_D, δg and ω is implied. This implies (2.20) when $C^{(0)} = 0$.[7] In general, terms of the form $\psi^2\hat{\partial}X$ arise from NS-NS flux and from the couplings of the worldsheet fermions to the spacetime affine connection. The linear combination in (2.20) is precisely that needed to reproduce the particular holomorphic index structure shown in (2.26). We will describe this relation in more detail below when we discuss $D_{\pm\pm}$ in terms of σ-model couplings. The arguments are essentially identical for D_{--}.

Next we consider the RR auxiliary field D_{+-}. The action of the spacetime supersymmetry charges on the vertex operator for D_{+-} is:

$$\left\{\hat{Q}^{(-1/2)}_{\dot{\alpha}}, \left[Q^{(-1/2)}_{\dot{\beta}}, V^{(-1/2,-1/2)}_{D_{+-}}\right]\right\} = \epsilon_{\dot{\alpha}\dot{\beta}} V^{(-1,-1)}_{w}\,, \tag{2.27}$$

where the superscripts denote the picture with respect to the gauged $\mathcal{N}=1$ worldsheet superconformal algebra. One may check that, at large volume and complex structure, the following operator has the correct transformation properties:

$$V^{(-\frac{1}{2},-\frac{1}{2})}_{D_{+-}} = g_s e^{-\frac{\phi}{2}-\frac{\hat{\phi}}{2}} \epsilon^{\alpha\beta} S_\alpha \hat{S}_\beta S^{-,\dot{a}} (C\Gamma)^{\bar{i}jk}_{\dot{a}\dot{b}} \hat{S}^{-,\dot{b}} \omega_{\bar{i}jk} + \cdots . \tag{2.28}$$

Here S_α is the positive chirality spin field for the $4d$ spacetime CFT; $S^{-,\dot{a}}$ is a negative chirality $6d$ spinor transforming as a $\bar{\mathbf{4}}$ under the SO(6) acting locally on the tangent space to X; $\Gamma^{ABC\cdots}$ are antisymmetrized products of the 6d Γ-matrices with indices in $\mathbf{4}\oplus\bar{\mathbf{4}}$; and C is a charge conjugation matrix intertwining the $\mathbf{4}$ and $\bar{\mathbf{4}}$ representations.

The vertex operator in (2.28) is clearly g_s times the vertex operator for a harmonic RR 3-form field strength $F \propto \omega$; this verifies (2.20) when $C^{(0)} = 0$.[8] The additional factor of g_s is needed for V_w to have the right normalization.

We should make some cautionary remarks at this point. One may only make small deformations of magnetic fields if there are noncompact three-cycles on which these fields have support. Otherwise, the fluxes satisfy quantization conditions, and small deformations that are constant in spacetime are not on-shell modes. (Although, as in [28], one may have solutions which interpolate in four dimensions between different values of magnetic 3-form flux.) Furthermore, in the case where the flux threads noncompact cycles, one must still take care with the vertex operators. In general they may have logarithmic OPEs with

[7] Similar results for the auxiliary fields in the gravity multiplet of $\mathcal{N}=1$ heterotic compactifications can be found in [51, 52].

[8] Recall that in string frame, the spacetime action for the RR fields is independent of g_s, so the corresponding vertex operator should also be independent of g_s.

themselves, due to their behavior at infinity in field space. A cautionary example in this regard is the open string vertex operator for a constant magnetic field strength on a bosonic D-brane in flat space:[9]

$$V = B_{\mu\nu} X^\mu \partial X^\nu + \cdots . \tag{2.29}$$

The term shown is not quite a scaling operator. Note that such pieces are missing from the analysis above, as they have nonsingular OPEs with the spacetime supercharges.

Auxiliary fields as sigma model couplings. Another way to understand the physics of the auxiliary fields is to ask, in worldsheet language, what sigma model couplings will break $\mathcal{N}=2$ spacetime SUSY to $\mathcal{N}=1$ spacetime SUSY, and which $\mathcal{N}=1$ subgroup will be preserved. Of course, in the RNS formalism, we will be restricted to considering D_{++} and D_{--} as we do not know how to treat nontrivial Ramond-Ramond backgrounds in the RNS formalism.

For example, if the auxiliary field D_{--} has an expectation value, the supersymmetry corresponding to $\hat{\theta}$ is broken; in particular, $\hat{\zeta}$ becomes a goldstino for this supersymmetry,

$$\hat{\delta}\hat{\zeta}_\alpha = \hat{\epsilon}_\alpha D_{--} . \tag{2.30}$$

Therefore, a vev for D_{--} will break the $\mathcal{N}=1$ supersymmetries arising from the right-moving sector of the worldsheet, but preserve the left-moving sector. The worldsheet theory must have $\mathcal{N}=(2,1)$ supersymmetry; the $\mathcal{N}=2$ supersymmetry for the left-movers leads to an $\mathcal{N}=1$ spacetime supersymmetry [53, 54], and the $(0,1)$ part of the worldsheet supersymmetry is gauged. Similarly, if we break the supersymmetries arising from the left-moving supercharges, the worldsheet theory should have $\mathcal{N}=(1,2)$ supersymmetry.

General sigma models with $\mathcal{N}=(2,1)$ supersymmetry were described by Hull [55]. These models contain non-vanishing 3-form field strengths which couple to the worldsheet fermions. Such field strengths couple to the left- and right-moving fermions with opposite sign: the quadratic terms in the lagrangian are:

$$L = -i\psi^\mu_\pm \left[g_{\mu\nu}\partial_\mp \psi^\nu_\pm \pm \left(g_{\mu\nu}\Gamma^\nu_{\lambda\rho} \pm H_{\mu\nu\rho} \right) \partial_\mp \phi^\lambda \psi^\rho_\pm \right] \tag{2.31}$$

$\mathcal{N}=2$ worldsheet supersymmetry in either the left- or right-moving sector requires a complex structure covariant with respect to the connection

$$\gamma^\mu_{\nu\rho,\pm} = \Gamma^\mu_{\nu\rho} \pm H^\mu_{\nu\rho} . \tag{2.32}$$

The result is that the metric g should be hermitean, the three-form field strength should be a $(2,1)$-form, and that the metric and NS-NS field strength should satisfy the equations:

$$H_{ij\bar{k}} = \pm i \left(\partial_j g_{i\bar{k}} - \partial_i g_{j\bar{k}} \right) = \pm T_{\bar{i}jk} . \tag{2.33}$$

This is consistent with (2.20). If $H = T$, then the left-moving supersymmetry is broken and the right-moving supersymmetry is intact, since $D_{++} \neq 0$, $D_{--} = 0$. If we choose the opposite sign in (2.33), the roles of D_{++}, D_{--} are reversed.

[9] We thank M. Berkooz for pointing this out to us.

Auxiliary fields for periods of fluxes. If one chooses the moduli space coordinates to be the periods t^a, then the auxiliary components of the resulting supermultiplets are:

$$\begin{aligned} D^a_{++} &= \int_{A^a} \left(\tilde{T} + \tilde{H}\right) \\ D^a_{+-} &= g_s \int_{A^a} \left(\tilde{F} - C^{(0)}\tilde{H}\right) \\ D^a_{--} &= \int_{A^a} \left(\tilde{T} - \tilde{H}\right). \end{aligned} \tag{2.34}$$

Eq. (2.34) can be demonstrated using (2.20) as follows. (Here we have restored the dependence on $C^{(0)}$, as we will justify below.) One can write the periods in terms of CFT coordinates, as implied by eq. (2.15). Now if V is an $\mathcal{N} = 2$ chiral superfield of the form (2.2), $t^a(V)$ is also a chiral superfield. Its lowest component is just the period $t^a(w)$. To find the auxiliary field, we need merely expand $t^a(V)$ to second order in $\theta, \hat{\theta}$. Let us compute the coefficient D^a_{++} of θ^2. We get this by expanding t to first order in the auxiliary fields D^m_{++} for w^m:

$$D^a_{++} = D^m_{++} \frac{\partial t^a}{\partial w^m}\,. \tag{2.35}$$

Now, using (2.15), and choosing a gauge such that $k_m = 0$ at the point in moduli space of interest, we find:

$$D^a_{++} = \sum_m \int_{A^a} (\tau^m + h^m)\omega_m = \int_{A^a} (\tilde{T} + \tilde{H})\,. \tag{2.36}$$

The computation for the other auxiliary fields is nearly identical, so (2.34) indeed follows from (2.20). If one chooses instead the dual coordinates F_a, then an analogous computation shows that

$$\begin{aligned} D_{++,a} &= \int_{B_a} \left(\tilde{T} + \tilde{H}\right) \\ D_{+-,a} &= g_s \int_{B_a} \left(\tilde{F} - C^{(0)}\tilde{H}\right) \\ D_{--,a} &= \int_{B_a} \left(\tilde{T} - \tilde{H}\right) \end{aligned} \tag{2.37}$$

It will be useful in section 3 to observe that these facts (2.34)(2.37) can be summarized by a 'three-form superfield' of the form

$$\Omega(\theta, \hat{\theta}) = \Omega + \theta^2 \left(\tilde{T} + \tilde{H}\right) + \hat{\theta}^2 \left(\tilde{T} - \tilde{H}\right) + \theta\hat{\theta} \left(\tilde{F} - C^{(0)}\tilde{H}\right) + \cdots. \tag{2.38}$$

D-branes and auxiliary fields. While it is difficult to discuss vevs for RR fields from the worldsheet in RNS language, we can examine spacetime solutions which have RR flux and $\mathcal{N} = 1$ supersymmetry, in order to confirm our identifications (2.20), (2.34), (2.37).

Let us start with a large number of space-filling D-branes in type-IIB string theory, wrapped on a holomorphic cycle so as to preserve $\mathcal{N} = 1$ supersymmetry in four dimensions. Such D-branes preserve the supersymmetry generated by $Q_\alpha + e^{i\gamma}\hat{Q}_\alpha$ for some phase $e^{i\gamma}$. For a single brane γ may be set to unity by redefining the phase of $\hat{Q}$. But if other branes and fluxes are present which also by themselves break $\mathcal{N} = 2$ SUSY to an $\mathcal{N} = 1$ subalgebra defined by a different phase, the relative phases will matter.

Brane-flux duality [31]–[32] states that the string background with D-branes is dual to a geometric background with nontrivial Ramond-Ramond flux. A particular solution describing D5-branes wrapped around a holomorphic 2-cycle is given in [56] when $C^{(0)} = 0$. Using this fact, the authors of [27, 30] have argued that one should set D_{++}, D_{+-}, and D_{--} all equal to F in order to preserve the same spacetime supersymmetry as the D-brane. This entails preserving the supersymmetry $Q + \hat{Q}$, which is precisely the supersymmetry preserved by a D5-brane wrapping a holomorphic cycle at large volume.[10] These authors identify $D_{\pm,\pm}$ with NS-NS flux. Our identification of auxiliary fields implies that $H = 0$, $T = g_s F$.[11] This identification of NS-NS auxiliary fields solves a slight puzzle in comparing the discussion of [27, 30] to the spacetime solution in [56], as the solution in [56] contains no NS-NS magnetic flux: the relevant NS-NS field is the three-form T built from the complex structure.

The S-dual solution in [56] describes an $NS5$-brane wrapping the same cycle. In this case, we can still preserve four supercharges, but there is no source for an RR field. Such solutions should therefore correspond to either D_{++} or D_{--} being nonzero, while the other vanishes. Which vanishes depends on whether one wraps a fivebrane or an anti-fivebrane around the cycle. The anti-fivebrane is a source for 3-form flux with sign opposite to the flux generated by a fivebrane. Therefore, if the Maldacena-Nuñez solution for a fivebrane is such that $H = T$, leading to $D_{--} = 0$, the solution for an anti-fivebrane has $-H = T$, leading to $D_{++} = 0$.

We can appeal to spacetime arguments to ask about the auxiliary fields when $C^{(0)}$ is nonvanishing. The simplest argument for the appearance of the $C^{(0)}$-dependent terms in (2.20), (2.34), and (2.37) is to notice how the spacetime spinors transform with F in type IIB [57, 58]. One can see directly from the string-frame presentation in appendix B of [58] that F always appears in the combination $F - C^{(0)}H$.

Finally, in compact models with branes and fluxes, one can turn on a combination of NS-NS and RR fluxes to maintain supersymmetry in the presence of nonvanishing D3-branes. The supersymmetry preserved by D3-branes is $Q + i\hat{Q}$. If we set $T = 0$, we can expand the vector multiplets out in powers of $\theta \pm i\hat{\theta}$. Choosing coordinates equal to periods t of the CY, we find:

$$V = t + ig_s(\theta + i\hat{\theta})^2\tilde{G} - ig_s(\theta - i\hat{\theta})^2\tilde{G}^* + \cdots \tag{2.39}$$

where $\tilde{G} = \tilde{F} - \tau\tilde{H}$, and $\tau = C^{(0)} + \frac{i}{g_s}$, and the tildes denote the components of the forms lying in $H^{(2,1)}(X)$. Note that if $G \in H^{(2,1)}(X)$, $G^* \in H^{(1,2)}(X)$, and $\tilde{G}^*$ vanishes. Therefore, we can see directly from the form of the auxiliary fields that an expectation value for G lying entirely in $H^{(2,1)}(X)$ preserves the same supersymmetry as a D3-brane placed in the CY background, consistently with [59, 60, 61, 62].

[10] The boundary conditions on the fermions are $\psi = -\hat{\psi}$ for worldsheet superpartners to Dirichlet directions and $\psi = \hat{\psi}$ for superpartners to Neumann directions. One may use this to deduce the action of the boundary conditions on the spectral flow operator, which at large volume can be constructed from the RNS fermions by bosonization.

[11] This value of T can be inferred from S-duality: the S-dual NS-NS solution on [56] must satisfy $H = \pm T$ in order to preserve $\mathcal{N} = 1$ SUSY, as we have discussed earlier.

2.2 Massless hypermultiplets

In type-IIB string theory, the Kähler moduli of the Calabi-Yau live in massless hypermultiplets. For a given element ω^a of $H^{(1,1)}(X)$, the four scalars correspond to the metric perturbation $g_{i\bar{j}} = g\omega^a_{i\bar{j}}$, the NS-NS two-form potential $b_{i\bar{j}} = b\omega^a_{i\bar{j}}$, the RR two-form potential $c_{i\bar{j}} = c\omega^a_{i\bar{j}}$, and a scalar which is the 4d dual of the RR 4-form potential $C^{(4)}_{\mu\nu i\bar{j}} = c_{\mu\nu}\omega^a_{i\bar{j}}$. These should form a triplet and a singlet under the SU(2) R-symmetry.

Ideally we would embark on a discussion of the auxiliary fields for IIB hypermultiplets. However, as we will see, these fields, as well as the NS-NS auxiliary fields in IIA vector multiplets, correspond to "mirrors of NS flux" [27, 43, 44], and are not yet well-understood. Therefore we will confine our discussion of hypermultiplets to type-IIA string theory compactified on CY threefolds, leaving the IIB discussion for future work. Assuming a suitable generalization of mirror symmetry can be formulated, the structure of auxiliary fields for hypers which we find in IIA will also govern the IIB physics.

In the hypermultiplets of type-IIA string theory, the complex structure deformations again play a starring role. There are four real scalars in this multiplet. Two derive from the complex scalar corresponding to deformations of the complex structure. The other two derive from the RR four-form field strength $F^{(4)}$. If ω^m is a basis element of $H^{(2,1)}$ corresponding to a complex structure deformation, then b^{21} complex vector field strengths F^m_μ arise via:

$$F^{(4)} = \sum_m F^m_\mu \omega^m_{\bar{i}jk} dx^\mu \wedge d\bar{z}^{\bar{i}} \wedge dz^j \wedge dz^k + \text{h.c.} \tag{2.40}$$

The Bianchi identities for $F^{(4)}$ and the fact that ω^M is a closed form imply that F is the derivative of a scalar.

$$F^m_\mu = \partial_\mu \phi^m \tag{2.41}$$

Thus we have two complex scalar fields for each $(2,1)$-form.

We can write a scalar superfield $\mathcal{H}$ for the hypermultiplet which satisfies the "twisted chiral" constraints:

$$\begin{aligned} \bar{\nabla}_{\dot\alpha}\mathcal{H} &\equiv \left(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\sigma^\mu_{\beta\dot\alpha}\theta^\beta\partial_\mu\right)\mathcal{H} = 0 \\ \hat{\nabla}_\alpha\mathcal{H} &\equiv \left(\frac{\partial}{\partial\hat\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\hat{\bar\theta}^{\dot\beta}\partial_\mu\right)\mathcal{H} = 0\,. \end{aligned} \tag{2.42}$$

The superspace expansion for the massless hypermultiplets is [13]:

$$\begin{aligned} \mathcal{H}^a &= w^a + \theta^\alpha\chi^a_\alpha + \hat{\bar\theta}^{\dot\beta}\hat{\bar\chi}^a_{\dot\beta} + \theta^2 y^a + \hat{\bar\theta}^2\hat{\bar y}^a + \theta^\alpha\hat{\bar\theta}^{\dot\beta}\sigma^\mu_{\alpha\dot\beta}F^a_\mu + \\ &+ \theta^\alpha\hat{\bar\theta}^2\eta^a_\alpha + \hat{\bar\theta}^{\dot\beta}\theta^2\hat{\bar\eta}^a_{\dot\beta} + \theta^2\hat{\bar\theta}^2 C^a \end{aligned} \tag{2.43}$$

Here w, F_μ, C are all complex. On-shell, w can be identified with a deformation of the complex structure and $F_\mu = \partial_\mu\phi$ is the corresponding RR axion scalar field strength.

All of the worldsheet arguments given for the type-IIB vector multiplets apply here in identifying $y, \hat{\bar y}$. While $\hat{\bar y}$ is related to w by $\hat{\bar Q}$ in type IIA, the worldsheet current $\hat{\bar Q}(\hat{z})$

for the spacetime supercharge depends on the same spectral flow operator, with $\mathrm{U}(1)_R$ charge 3/2, as the current $\hat{Q}$ does in type-IIB string theory. Therefore, if we use "CFT coordinates" we can identify:

$$y^m = \tau^m + h^m \hat{y}^m = \tau^m - h^m \,, \tag{2.44}$$

where τ^m, h^m are given in (2.19). Similarly, if we choose the periods of the (3,0) form as our moduli space coordinates, we find that:

$$\begin{aligned} y^a &= \int_{A^a} \left(\tilde{T} + \tilde{H}\right) \\ \hat{y}^a &= \int_{A^a} \left(\tilde{T} - \tilde{H}\right). \end{aligned} \tag{2.45}$$

and if we choose the dual periods:

$$\begin{aligned} y_a &= \int_{B_a} \left(\tilde{T} + \tilde{H}\right) \\ \hat{y}_a &= \int_{B_a} \left(\tilde{T} - \tilde{H}\right). \end{aligned} \tag{2.46}$$

Note that the mixed $\theta\hat{\theta}$ term in (2.43) is a propagating RR axion. The corresponding vertex operator for this field is:

$$V^{(-\frac{1}{2},-\frac{1}{2})}_{F^m_\mu} = e^{-\frac{\phi}{2}-\frac{\hat{\phi}}{2}} F^m_\mu S^\alpha \sigma^\mu_{\alpha\dot{\beta}} \hat{S}^{\dot{\beta}} S^{-,\dot{a}} (C\Gamma)^{ij\bar{k}}_{\dot{a}\dot{b}} \hat{S}^{-,\dot{b}} \omega^m_{ij\bar{k}} \,. \tag{2.47}$$

Again, $S^\alpha, \hat{S}^{\dot{\beta}}$ are spin fields describing 4d spinors, while $S^{-,\dot{a}}, \hat{S}^{-,\dot{b}}$ are spin fields for internal $6d$ spinors.

A final check of these identifications is the solution in [56] corresponding to NS5-branes wrapped on a holomorphic 2-cycle. This is a solution in both type-IIA and -IIB string theory, as it contains vevs for NS fields only, and it breaks either Q or $\hat{Q}$, depending on the relative sign of H and T.

2.3 Comparison to the hybrid formalism

Berkovits and Siegel [14, 13] have constructed a manifestly spacetime supersymmetric worldsheet theory for type-II and heterotic strings compactified on a Calabi-Yau background. In this formalism, the superspace coordinates $x, \theta, \hat{\theta}$ appear as worldsheet fields, and together with an additional boson ρ describe a CFT with a nonlinearly realized $\mathcal{N} = 2$ superconformal field theory. This SCFT is then combined with a (twisted) $c = 9$ $\mathcal{N} = 2$ superconformal field theory which is the usual CFT describing the Calabi-Yau compactification. The $\mathcal{N} = 2$ superconformal is gauged. There is no barrier to writing down the worldsheet theory in the presence of nonvanishing RR fields.

This appears to be an ideal formalism for the models discussed in this paper. Nonetheless, there remain some things to be understood. In [14, 13], the authors argue that the $\mathcal{N} = 2$ physical state constraints require that D_{ij} and y vanish. In view of the results

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of [31]–[32], and our identifications of the auxiliary fields, there appears to be a problem. Presumably the physical state constraints are modified in the presence of nonvanishing flux.[12] We will leave this question for future work.

3. Engineering soft breaking terms in string theory

Now that we have identified the auxiliary fields for the closed string moduli, we can ask how these fields appear in the low-energy effective action as coefficients of SUSY-breaking operators. We will start by reviewing the argument in [27] for the superpotential in [28, 29], and then discuss fluxes which break $\mathcal{N}=1$ supersymmetry completely.

3.1 The GVW superpotential

Given the representation of the vector multiplets in type-IIB string theory in terms of $\mathcal{N}=2$ chiral superfields V^a, we can write the low energy lagrangian for these fields locally in the vector multiplet moduli space in terms of a holomorphic prepotential $\mathcal{F}(t)$, such that the dual variables F_a can be written as $F_a = \partial_a \mathcal{F}$. The low-energy effective lagrangian can be written in terms of an unconstrained V and a chiral superfield V_D which acts as a Lagrange multiplier [63, 46]:

$$\mathcal{L} = \int d^2\theta d^2\hat{\theta} \left(\mathcal{F}(V) - \sum_a V_{D,a} V^a \right). \tag{3.1}$$

The equation of motion for V is

$$V_{D,a}(V) = \partial_a \mathcal{F}\,.$$

Integration over V_D implements the constraint (2.3) on V.

Vafa [27] has shown in a type-IIA model that when the auxiliary fields in V, V_D have nonvanishing vevs, one can expand (3.1) in these auxiliary fields and get the superpotential described in [28, 29]. For a proper choice of vev, the vevs of these auxiliary fields break $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$, as we have discussed in the previous section. For example, we can choose a flux $G \in H^{(2,1)}$, which leaves unbroken the supersymmetry that generates superspace translations along $\theta - i\hat{\theta}$. Indeed, upon integrating over $\theta + i\hat{\theta}$ in (3.1), one finds a $\mathcal{N}=1$ superpotential term which is linear in the auxiliary fields, and therefore the fluxes:

$$W = -D_D \cdot \mathcal{A}(x, \theta - i\hat{\theta}) \tag{3.2}$$

where $D_D = \int_B G$ is the auxiliary field multiplying $(\theta + i\hat{\theta})^2$, and $\mathcal{A}$ is the $\mathcal{N}=1$ chiral superfield one gets from translating V in the $(\theta - i\hat{\theta})$ direction of superspace:

$$\mathcal{A} = w + (\theta - i\hat{\theta})\zeta + (\theta - i\hat{\theta})^2 D\,. \tag{3.3}$$

It is known [28, 29] from other arguments that nonvanishing G induces a superpotential for complex structure moduli:

$$W(t) = \int G \wedge \Omega(t) = \sum_a \int_{A^a} G \int_{B_a} \Omega - \int_{B_a} G \int_{A^a} \Omega\,. \tag{3.4}$$

[12] We would like to thank N. Berkovits for correspondence on this issue, and for suggesting this resolution.

Eq. (3.2) has the right form, but it is missing the piece proportional to $\int_A G \int_B \Omega = F_a \int_A G$. This is as it should be. A spurion superfield, as used in [8], should be nondynamical: for these superfields, we can tune the vevs of the component fields by hand without going off shell. The auxiliary fields D_{ab} for V lie in supermultiplets for propagating particles. Vevs for these fields must arise from spontaneous SUSY-breaking, which arise in global $\mathcal{N}=2$ theories via Fayet-Iliopoulos terms [63, 64, 46].

We can give an explicit example where the superfields for the dual periods can be spurions, allowing a desciption of explicit soft SUSY breaking to $\mathcal{N}=1$. Following [32], choose the Calabi-Yau hypersurface

$$y^2 + u^2 + v^2 = W'(x)^2 + f(x) \tag{3.5}$$

where $W(x)$ is an $(n+1)$st-degree polynomial (so that $W'(x)$ is an nth-degree polynomial) and $f(x)$ is a degree $(n-1)$ polynomial. The complex structure moduli space has been described in e.g. [32]. There are n complex structure deformations which are normalizable, in the sense of having a finite kinetic term, and which can be described as deformations of f. These control the volumes of n compact, independent, nonintersecting cycles we will label A_a, localized near the zeros of $W' = \prod_{a=1}^{n}(x - x_a)$. There is a set of noncompact dual cycles B^a which intersect A_a once and extend to infinity.

The periods t^a along A_a are propagating vector multiplets, while the dual periods for B^a can be treated as spurions: they are clearly not normalizable, and the fluxes through these noncompact cycles are not quantized and may be tuned continuously from zero. In this case, the superpotential (3.2), which is the realization of (3.4) in this case, is linear in the periods of the A-cycles. The corresponding scalar potential will only vanish when the inverse metric on the moduli space vanishes, which occurs when the A-cycles shrink to zero volume [65].

For flux through compact cycles, we can explicitly identify fields which control the FI parameters. For illustration, focus on a single pair of compact A and B cycles. A D3-brane on a special lagrangian representative of the homology class of the B-cycle is a hypermultiplet which is *magnetically* charged under the vectormultiplet built on t. Its mass is determined by the volume of the B-cycle $m_Q = F_t = \int_B \Omega$. Call the scalars in this multiplet $Q_i, i = \pm$. $\mathcal{N}=2$ supersymmetry implies a superpotential coupling $W_{\mathcal{N}=2} = Q_+ F_t Q_-$ which encodes the mass. Further, the magnetic gauge field (the gauge field associated to the B-cycle) is related by $\mathcal{N}=2$ SUSY to a triplet of auxiliary fields D_{Dij}, and the electric charge of Q under this gauge field implies a coupling to the scalars of the form

$$Q_i Q_j D_{Dij} \,.$$

Therefore, a vev

$$\langle Q_i Q_j \rangle = r_{ij}$$

acts precisely as the magnetic FI coupling for this vectormultiplet rD_D. Plugging this into the $\mathcal{N}=2$ lagrangian and varying D_D, one learns that the vev of the auxiliary field in the electric vectormultiplet is

$$D_{ij} = r_{ij} \,.$$

We have identified D with the flux through the A-cycle. It will be interesting to understand the origin of flux quantization from this perspective.

3.1.1 Effect of fluxes on wrapped branes which are particles

Before continuing, we would like to discuss how such fluxes affect the physics when D-branes wrapping vanishing cycles enhance the closed-string gauge group to a nonabelian gauge group. In this case the wrapped branes are charged under the vector multiplets controlling the periods of the cycles that are wrapped. Branes wrapping dual cycles are magnetically charged. However, the background fluxes do not allow charged states made out of these wrapped branes. If we make a particle state by wrapping a Dp-brane on a p-cycle W which has p-form RR flux through it, then because of the worldvolume coupling

$$\int_{Dp} F \wedge C_{p-1} = -\int_{Dp} A \wedge F_p \,,$$

where $F = dA$ is the worldvolume gauge field, the Gauss' law on the Dp-brane is modified to

$$0 = \frac{\delta S}{\delta A_0} = \int_W F_p + \delta(\partial F1) \,. \tag{3.6}$$

The brane can't solve its own Gauss' law unless it has the right number $q = \int_W F_p$ of F-strings ending on it [65, 66]. Therefore, such branes must come in pairs comprised of a brane and an anti-brane, with q F-strings stretching between them.

Furthermore, the flux through the 3-cycle will try to prevent that cycle from shrinking. This is clear on general energetic grounds, and is borne out by extremizing the GVW superpotential. Both of these effects are in harmony with what is known about the low energy effective action in the presence of such a flux. As pointed out in [29] in the context of type-IIA string theory, such a flux corresponds precisely to breaking $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$ by giving a mass to the scalar in the closed-string vector multiplet, while branes wrapping the dual cycles will be light magnetic monopoles, with a mass controlled by the dual period. Thus, the terms in [29] that arise from the flux are known to lead to monopole condensation and confinement [67, 68].

3.2 Soft breaking parameters through F-terms

Gauge theories with superpotentials for adjoint scalars are easy to realize in type-IIB string theory [33, 69]. As an example, begin with D5-branes wrapping holomorphic 2-cycles in a Calabi-Yau manifold. There are adjoint chiral superfields arising from holomorphic deformations of the supersymmetric cycle and of open string gauge bundles living on those cycles. To all orders in perturbation theory, the superpotential for these fields is determined entirely by the obstructions to finite holomorphic deformations of these cycles, and so the superpotential couplings depend entirely on complex structure moduli [33, 34, 70]. In other words, the superpotential can be written as

$$W = W(t_a, \Phi) \tag{3.7}$$

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where t_a are the complex structure moduli. At energies low enough that gravity decouples from the D-brane gauge theory, t_a may be treated as couplings. Supersymmetry is broken if these couplings are treated as chiral superfields and the auxiliary components of these superfields are given vacuum expectation values.

For example, for the quadratic and cubic terms in the superpotential,

$$W_{\rm rel} = \frac{1}{2} t_2 \,{\rm tr}\, \Phi^2 + \frac{t_3}{3!} \,{\rm tr}\, \Phi^3 \tag{3.8}$$

the SUSY-violating terms are soft and do not spoil the ultraviolet properties of the theory. If we promote $t_{2,3}$ to superfields $T_{2,3}$, the most general expectation value preserving Lorentz symmetry is:

$$T_i = t_i + \theta^2 F_i \,. \tag{3.9}$$

Following the discussion in section 2.1, F_i will correspond to a vev for the flux $\bar{G} \in H^{(2,1)}(X)$, or equivalently a vev for $G \in H^{(1,2)}(X)$. This breaks the spacetime SUSY unbroken by the D5-brane (at large volume).

The action which derives from (3.8) is:

$$\int d^4x \, d^2\theta \, W(T, \Phi) + \text{h.c.} = \int d^4x \, d^2\theta \, W(t, \Phi) + \text{h.c.} + S_{sb} \tag{3.10}$$

where the SUSY-breaking terms are:

$$S_{sb} = \int d^4x \left(F_2 \,{\rm tr}\, \phi^2 + F_3 \,{\rm tr}\, \phi^3 + \text{h.c.} \right) . \tag{3.11}$$

The SUSY-breaking part of the action includes only the scalar components of Φ. ϕ has two real components and the mass term F_2 induces a mass matrix with positive and negative mass squared. F_3 induces a SUSY-breaking Yukawa coupling.

We can give a partial worldvolume argument for these SUSY-breaking terms. A D5-brane couples minimally to the RR 6-form potential, $S_{D5} \ni \int_{D5} C^{(6)}$. We are interested in a D5-brane which wraps a curve in the CY and fills the transverse R^4. With this in mind, decompose the six-form potential as $C^{(6)} = dv \wedge c^{(2)} + \cdots$ where $dv = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ and $\cdots$ indicates other polarizations which will not concern us. The expression of minimal coupling above is shorthand for the 4d lagrangian density

$$L_s = \int_{\mathcal{C}} {\rm tr}\, \Phi^\star c^{(2)} \,; \tag{3.12}$$

following [71], we have described the part of the worldvolume of the brane in the CY as an embedding from an abstract curve $\mathcal{C}$ into the CY. In this description, the RR potential must be pulled back to the branes by the embedding fields Φ, and the trace indicates that this pulled-back potential is really a matrix fuction of the matricized embedding coordinates. Next, extend the map Φ to a three-chain Ξ with which bounds the curve $\mathcal{C}$: $\partial\Xi = \mathcal{C}$. The coupling (3.12) can be rewritten as an integral over the bounding 3-chain, as

$$L_s = \int_{\Xi} \Phi^\star g^{(3)} \,, \tag{3.13}$$

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where $g = dc$. By self-duality of RR fluxes in type IIB, the seven-form flux satisfies

$$F^{(7)} = dv^{(4)} \wedge g^{(3)} = \star F^{(3)} \,. \tag{3.14}$$

Therefore, $g^{(3)} = \star_{CY} F^{(3)}$, and the induced bosonic potential can be written as:

$$L_s = \int_\Xi \Phi^\star \left(\star_{CY} F^{(3)} \right) . \tag{3.15}$$

In the presence of a constant RR axion $C^{(0)}$, the Witten effect modifies the minimal coupling to $S_{D5} \ni \int_{D5} \left(C^{(6)} - C^{(0)} B^{(6)} \right)$ where $dB^{(6)} = \star H$ is the field to which NS5-branes couple electrically. Retracing our steps, (3.15) becomes $\int_\Xi \Phi^\star \left(\star_{CY} \left(F - C^{(0)} H \right) \right)$. Let us consider for the moment $C^{(0)} = 0$. Let us turn on the RR 3-form potential such that $F = F^{(2,1)} + F^{(1,2)}$. The terms on the right hand side are imaginary self dual and imaginary anti-delf-dual, respectively. Therefore,

$$\star F = i(F^{(2,1)} - F^{(1,2)}) \,, \tag{3.16}$$

and the potential is:

$$\int_\Xi \Phi^\star \left(iF^{(2,1)} - iF^{(1,2)} \right) . \tag{3.17}$$

Compare this coupling with Witten's description [71] of the obstruction *super*potential as a function of the embedding fields Φ and the holomorphic threeform $\Omega(t)$:

$$W(t, \Phi) \propto \int_\Xi \operatorname{tr} \Phi^\star \Omega(t) \,. \tag{3.18}$$

Now let $\eta = \theta + \hat{\theta}, \xi = \theta - \hat{\theta}$. η corresponds to the supersymmetry which is preserved by the D5-branes. We may rewrite (2.38) as:

$$\Omega(\eta, \xi) = \Omega(t) + \eta^2 (\tilde{F} + \tilde{T}) + \xi^2 (\tilde{F} - \tilde{T}) + \cdots \,. \tag{3.19}$$

Turning on F without turning on T clearly breaks the remaining SUSY preserved by the D5-brane. Integrating (3.18) along the superspace directions preserved by a D5-brane, using the identifications of section 2, one indeed finds that the potential (3.17) matches the SUSY-breaking potential derived from (3.18).

3.2.1 An example

The canonical example of a curve with obstructed deformations is found in the following hypersurface singularity in $\mathbb{C}^3$:

$$u^2 + v^2 + y^2 = x^{2n} \,. \tag{3.20}$$

A small resolution of this singularity introduces a homologically nontrivial $\mathbb{P}^1$ at the origin of $\mathbb{C}^3$. The normal bundle of this curve has a holomorphic section with an nth order obstruction [72]. N D5-branes wrapped around such a cycle have as their low-energy degrees of freedom a $\mathrm{U}(N)$ gauge field and an adjoint scalar Φ with superpotential $W = \Phi^{n+1}$ [33, 69, 32].

Let us choose $n = 2$ to get a cubic superpotential. A mass term is turned on if one deforms (3.20) so as to split the singularity:

$$u^2 + v^2 + y^2 = (x^2 - a^2)^2 . \tag{3.21}$$

The geometry has a compact S^3 which can be described as follows.

For each x, (3.21) describes an A_1 ALE space. This space has a resolved S^2 with radius $r = |x^2 - a^2|$, described by a real slice of (3.21) at fixed x. If we fiber this S^2 over the line between $x = a$ and $x = -a$, we sweep out an S^3 which we will denote as C [73]. Following the discussions in [32, 17], there are two noncompact "B-cycles" which are the 2-spheres in the (y, u, v) directions fibered over lines running from $x = \pm a$ to infinity, and which we will label $B_\pm$.

We can resolve the conifold singularities at $x = \pm a$ with normalizable deformations of (3.21) as follows:

$$u^2 + v^2 + y^2 = (x^2 - a^2)^2 + bx + c \tag{3.22}$$

where b, c are the normalizable deformations. These split the double points at $x = \pm a$, introducing finite size three-cycles $A^\pm$ which are dual to $B^\pm$ and which intersect C with intersection number ± 1.

On the other hand, if we perform a small resolution, we introduce one $\mathbb{P}^1$ at each of $x = \pm a$. These are homologous. C becomes a three-chain whose boundary is the difference of these two $\mathbb{P}^1$s, and $B^\pm$ become three-chains as well. The parameter a is a nonnormalizable complex structure parameter which controls the distance between the two rational curves at $x = \pm a$.

If we perform a such a small resolution and wrap a D5-brane around one of the $\mathbb{P}^1$s, there will be a single scalar field parameterizing holomorphic deformations of the $\mathbb{P}^1$, and the superpotential will take the form [33, 69]:

$$W = \mathrm{tr}\left(\frac{1}{3}\Phi^3 - a^2\Phi\right). \tag{3.23}$$

By changing variables to $\tilde{\Phi} = \Phi + a$, and expanding W mear $\tilde{\phi} = 0$, one can see that a is a mass parameter. The SUSY-breaking flux that F_a corresponds to will be $\bar{G}$ with support on the three-cycles whose periods depend on a. Such fluxes are three-forms which do not die off at infinity. We can understand better how these can arise when we embed (3.21) in a more complete model.

3.2.2 Three-cycles for nonnormalizable deformations

Let us explain how (3.21) describes a patch of a slightly more global description of a compact geometry. Let Y be a genus-g curve $\mathcal{S}_g$ of singular A_1 ALE fibers embedded in some Calabi-Yau [74, 75]. A particular example [74] is the genus-three curve of A_1 singularities at $z_1 = z_2 = 0$ in the hypersurface

$$z_1^8 + z_2^8 + z_3^4 + z_4^4 + z_5^4 + \cdots \tag{3.24}$$

in $\mathbb{CP}_{1,1,2,2,2}$. We can now blow up the orbifold singularity, giving us a family S_g of $\mathbb{P}^1$s. D5-branes wrapped around an $\mathbb{P}^1$ fiber have a moduli space which is this Riemann surface [76].

However, there is a set of non-toric (in this realization) complex structure deformations which can be related to harmonic forms $\omega \in H^1(S_g)$. Turning on such deformations leave in general $(2g-2)$ rational curves at points on S_g corresponding to zeros of ω. Now, we can describe a star-shaped patch of this Riemann surface by a complex coordinate x. Locally near some set of n zeros of $\omega(x)$, this can be modeled precisely by (3.5) with $f(x) = 0$, and D5-branes have a superpotential described in $W'(x)$, as can be inferred from [76]. The nonnormalizable deformations in (3.5) are in fact the "non-toric" deformations of Y. Furthermore, one may blow down the A_1 fibers and pass through an extremal transition. In the local model this can be described via a deformation $f(x)$ in (3.5).

A natural set of three-cycles E^i corresponding to the non-toric deformations are fibrations of S^2s in the A^1 fiber over the cycles of the Riemann surface $\mathcal{S}_g$ [74, 75]. From the point of view of the local patch these three-cycles exist because of nontrivial structure hidden at infinity. In this example, the periods of C, and $B^{\pm}$ and some of E^i all depend on a.

3.3 Gaugino mass terms

The kinetic term for an $\mathcal{N} = 1$ vector can be written as a half-superspace integral

$$\mathcal{L} = \int d^2\theta \, \tau_{\rm YM}(\phi) W_\alpha W^\alpha \tag{3.25}$$

where $\tau_{\rm YM}(\phi)$ is the chiral superfield gauge coupling function. From (3.25) we see that a mass for this gaugino results from giving vev to the θ^2 component of the chiral multiplets on which $\tau_{\rm YM}$ depends:

$$m_{\rm gaugino} = F^a \partial_a \tau_{\rm YM} \tag{3.26}$$

where F^a is the auxiliary field corresponding to ϕ.

Which closed string fields appear in $\tau_{\rm YM}$ depends on how the gauge symmetry is realized. For space-filling B-type D-branes in type IIB, the gauge coupling is controlled, to leading order, by the volume of the cycle the brane wraps, which is in turn controlled by the Kähler moduli. These live in hypermultiplets. Therefore, the gaugino masses will depend on the auxiliary fields $y, \hat{y}$. We understand these fields somewhat better in type-IIA models. The mirror D-brane configuration is a D6-brane wrapped on a special lagrangian 3-cycle $A \subset X$ with phase $e^{i\gamma}$. The gauge coupling is a function of the volume of this cycle, and is in fact linear in this volume to leading order. The tree level gauge coupling function is

$$\tau_{\rm YM} = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{\rm YM}^2} = \frac{i}{g_s} \mathrm{Re}\, e^{i\gamma} \int_A \Omega + \int_A C \tag{3.27}$$

The gaugino masses will arise from the auxiliary field for t. For example, if we break the SUSY charge $\hat{Q}$, the gaugino mass is proportional to

$$m_{\rm gaugino} = \mathrm{Re}\, e^{i\gamma} y \tag{3.28}$$

where $y = \int_A (H - T)$ is the flux through the cycle on which the D6-brane is wrapped.

Another possible source of gauge dynamics is from vanishing cycles, if there are branes which can wrap these cycles and which live at a point in the four-dimensional spacetime. We will focus on singularities arising from D3-branes wrapped around vanishing 3-cycles in type-IIB string theory. If these branes have spin-1 states, they lie in vector multiplets which become massless when the geometry becomes singular. Away from the singularity, the gauge symmetry is completely broken down to the maximal torus of the original group. This maximal torus consists of the vectors in the vector multiplets that control the volume of the vanishing cycles, which are perturbative degrees of freedom in type-IIB string theory. The dynamics of these vector multiplets are controlled by the perturbative prepotential.

The difference from the previous example is that these gauge symmetries arise already in the $\mathcal{N} = 2$ theory before SUSY was broken. Therefore, the gauge dynamics is a function of the vector multiplets, in contrast with the wrapped-brane example above.

The gauge coupling of the $\mathcal{N} = 2$ theory is the second derivative of the prepotential:

$$\tau_{ab} = \mathcal{F}_{ab}(V) \tag{3.29}$$

Choose for example a single abelian vector multiplet V with a scalar t equal to the period of a 3-cycle A. A D3-brane wrapped on this three-cycle is charged under the vector multiplet. If it has a spin-one excitation, the perturbative U(1) gauge theory is enhanced to SU(2). The gauge coupling is

$$\tau_{\rm YM} = \mathcal{F}_{aa}(V)\,.$$

If we break the supersymmetry corresponding to $\hat{Q}$, t^a lives in a chiral multiplet with respect to Q that includes t^a and y^a, where we replaced w^a with t^a in (2.43). Hence $D^a_{--} = \int_A (H - T)$ is the gaugino mass.

3.4 Scalar mass terms

Another phenomenologically important SUSY-breaking mass term is of the type:

$$S_{sb} = \int d^4x\, M \phi^\dagger \phi \tag{3.30}$$

which gives a positive mass to both the real and imaginary parts of ϕ. In particular squark masses are of this form in the minimal supersymmetric standard model.

In general such masses arise can arise via spurions in two ways, depending on whether the spurion is a chiral or a vector multiplet. Imagine a Kähler potential for some scalar field ϕ, as a function of moduli coordinates t. If the spurion is a chiral multiplet, let the corresponding superfields be T, Φ. If T has as an auxiliary field F, a term in the Kähler potential of the form

$$K(t, \phi) = \frac{1}{\Lambda^2} T^\dagger T \Phi^\dagger \Phi \tag{3.31}$$

leads to a mass term for ϕ:

$$S_{sb} = \int d^4x\, \frac{|F|^2}{\Lambda^2} |\phi|^2\,. \tag{3.32}$$

For general open string fields, the Kahler potential can be a function of both vector and hypermultiplets, in which case fluxes can generically lead to squark masses.[13]

Similarly, we can imagine that the scalar field ϕ is charged under a nondynamical vector multiplet, so that the action is

$$S = \int d^4x\, d^4\theta\, \Phi^\dagger e^V \Phi \tag{3.33}$$

where V is a vector superfield. A vev for the auxiliary scalar D in this vector multiplet will induce a soft scalar mass $D|\phi|^2$. However, in general open strings will not be charged under closed string gauge groups. Such a term might arise if the squarks are charged under a weakly coupled U(1) propagating on a brane distinct from the branes carrying the standard model gauge symmetries. If SUSY is broken on this other brane, such a coupling may emerge.

4. The Dijkgraaf-Vafa conjecture and softly broken SUSY

Dijkgraaf and Vafa [17] have argued that the full set of F-terms in the low-energy effective action of $d = 4$, $\mathcal{N} = 1$ SUSY gauge theories can be computed using the saddle point approximation to an auxiliary matrix integral. The original motivation for this conjecture arose from the appearance of such field theories as the low-energy description of open strings ending on D-branes, in type-II string theory compactified on a Calabi-Yau threefold, precisely the class of theories we have discussed above. Since then, Dijkgraaf et al. have provided a perturbative derivation of this conjecture directly in the quantum field theory [77, 78], and Cachazo et al. [78] have given a nonperturbative argument via the Konishi anomaly.

These arguments might appear to fail when supersymmetry is broken dynamically or via explicit soft SUSY-breaking terms. Important elements of the proof outlined in [78] depend crucially on the fact that the SUSY charges annihilate the vacuum when SUSY is unbroken: the statements that correlators of gauge-invariant chiral fields are constant [79, 80, 81, 78], that they are holomorphic in the superpotential couplings, and that the vacuum expectation values of the non-chiral terms in the Konishi anomaly equation vanish.

We find that the Dijkgraaf-Vafa proposal for computing low-energy superpotentials retains some force when supersymmetry is broken via a class of explicit soft supersymmetry-breaking terms, if these terms are small compared to the other dynamical scales of the theory.[14] This follows earlier discussions of perturbative [8] and nonperturbative [15] non-renormalization theorems for F-terms when supersymmetry is softly broken. The essential point is that so long as one is asking holomorphic questions, one will get holomorphic answers. For this to be possible, one must be able to isolate the holomorphic parts of amplitudes. We may do this as long as the SUSY-breaking terms are genuinely soft, meaning that they are indeed relevant operators whose effects are negligible in the UV. In such

[13] The form of these couplings which arises in a supergravity approximation are determined in [23, 24].

[14] The paper [82] uses the techniques in [78] to examine the IYIT model [83] of dynamical supersymmetry breaking.

a circumstance, amplitudes will be analytic in the soft-breaking couplings, and one can isolate the holomorphic part by expanding in the antiholomorphic couplings and keeping the constant term.

Although one may compute the F-terms in these examples, one must face the fact that they no longer control the correlators of chiral operators; D-terms infect all of the answers and the effects are calculable only when the soft breaking terms are small. Furthermore, once the soft SUSY-breaking masses are larger than the dynamical scales of the theory, the results we extract using supersymmetry fail to be a useful guide to low-energy physics.

In the remainder of this section we will make these statements more precise. After quickly reviewing the Dijkgraaf-Vafa conjecture in the simplest case without SUSY breaking ($\mathrm{U}(N)$ gauge theory with an adjoint chiral scalar), we will demonstrate first the sense in which the perturbative proofs in [77, 78] remain valid when explicit soft SUSY breaking terms are added to the action, and then how the nonperturbative arguments of [78] remain valid.[15]

4.1 A brief review of the Dijkgraaf-Vafa conjecture

To understand the conjecture let us describe the simplest case, that of an $\mathcal{N} = 1$ $\mathrm{U}(N)$ gauge theory in four dimensions, coupled to an adjoint chiral superfield Φ. The gauge coupling and theta angle can be written as a complex coupling $\tau = 4\pi i/g^2 + \theta/2\pi$. Let Φ have a tree-level superpotential $W_0(\Phi)$; if the superpotential has k extrema and one chooses the vev of Φ such that N_i eigenvalues reside in the ith critical point, the gauge group is broken to:

$$\mathrm{U}(N) \longrightarrow \mathrm{U}(N_1) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_k)\,. \tag{4.1}$$

This theory confines, and the low-energy degrees of freedom are believed to be described by the "glueball" superfields

$$S_i = \frac{1}{32\pi^2}\,\mathrm{Tr}\,\mathcal{W}_{i,\alpha}\mathcal{W}_i^{\alpha}\,, \tag{4.2}$$

where $\mathcal{W}_{\alpha,i}$ is the fermionic chiral superfield for the vector multiplet of the unbroken gauge group $\mathrm{U}(N_i)$. Much of the nonperturbative information about the gauge theory is captured in the low-energy effective action for S_i [85].

The conjectured nonperturbative glueball superpotential is:

$$W(S_i) = \sum_i N_i \frac{\partial F_0(S)}{\partial S_i} + b^i S_i\,. \tag{4.3}$$

Here $F_0(S_i = g_s M_i)$ is the saddle-point solution to the free energy of the following holomorphic matrix integral:

$$Z = \int d\Phi e^{-\frac{1}{g_s}W_0(\Phi)} \sim e^{-\frac{1}{g_s^2}F_0(g_s M_i)}\,. \tag{4.4}$$

Here Φ is an $M \times M$ complex matrix. It can be written as $\Phi = U^\dagger \Lambda U$ where Λ is diagonal and U is unitary. The M_i, $\sum_i M_i = M$, are specified by running M_i of the eigenvalue

[15] See [84] for an early discussion of the low-energy effective action for a system with dynamical SUSY breaking.

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contours over the ith critical point of W_0. In the simplest example of the most symmetric vacuum, the integral over U factors out and leads to the volume factor [86, 87]

$$\int dU = M^{M^2/2} = e^{\frac{1}{2}M^2 \ln M} = e^{\frac{1}{g_s^2}S^2 \ln S} \tag{4.5}$$

Therefore, the volume factor contains the Veneziano-Yankielowicz term [85], from which the existence of N supersymmetric vacua [88] can be understood [85, 89] (for a review of this approach see chapter 8 of [81]).

4.2 Perturbative arguments

The perturbative argument for this result, enunciated in [77], and summarized in [78, 90], is phrased entirely in terms of superfield perturbation theory. In this language it is simple to introduce explicit soft SUSY breaking in the F-term sector, by treating the couplings of the theory as (chiral) spurion superfields, and letting the auxiliary components take nonzero values. We will focus on the illustrative example of the superpotential

$$W_0(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 \tag{4.6}$$

and replacing m with a chiral "spurion" superfield, so that:

$$m \longrightarrow M = m + \theta^2\Delta \tag{4.7}$$

in (4.6), the classical lagrangian is modified by a SUSY-breaking term which in component form is:

$$\delta\mathcal{L} = \int d^4x \left(\Delta \,\mathrm{tr}\, \phi^2 + \bar{\Delta}\,\mathrm{tr}\, \bar{\phi}^2\right). \tag{4.8}$$

All of the arguments of [77, 78] proceed as before; they are based on superfield perturbation theory, and we need merely replace m with M in each diagram.[16] The result of this can be seen by studying the leading correction to the Veneziano-Yankielowicz superpotential:

$$W(S) = NS\ln(S/\Lambda_0^3) + 2\pi i\tau S + 2N\frac{g^2}{m^3}S^2 + \cdots \tag{4.9}$$

Replacing m with M, and doing the superspace integral, the correction term to the Langrangian is linear in Δ:

$$L = L_{(\Delta=0)} - 6\frac{g^2}{m^4}\bar{\Delta}\sigma^2 + \text{h.c.} \tag{4.10}$$

where $S = \sigma + \sqrt{2}\theta\psi_S + \cdots$. This is a soft mass for the glueball scalar. This result is consistent with the string theory interpretation, where the glueball superfield is part of a closed-string vectormultiplet. This multiplet is associated to a 3-cycle which arises via a conifold transition from the 2-cycle which the 5-branes were wrapping. We have shown earlier that the soft-breaking parameter Δ arises as a mode of the flux through the cycle whose period is m. Using again the GVW superpotential, one finds a soft term of the form (4.9).

[16] Ref. [91] does precisely this, in order to compute the gaugino mass induced by soft breaking.

Although the F-terms behave simply, the explicit breaking ruins the holomorphicity properties of the theory. One can see this most simply by studying a single neutral chiral scalar multiplet Φ in four dimensions, with canonical kinetic terms, and with the same bare superpotential and soft SUSY-breaking term as above. To zeroth order in Δ, the chiral correlator for the scalar components ϕ vanishes:

$$\langle\phi(x_1)\phi(x_2)\rangle = 0 + \mathcal{O}(\bar{\Delta}) + \cdots \tag{4.11}$$

To first order in $\Delta, \bar{\Delta}$, the above correlator has a nonconstant antiholomorphic piece:

$$\langle\phi(x_1)\phi(x_2)\rangle = \langle\phi(x_1)\phi(x_2)\textstyle\int d^4x\bar{\Delta}\bar{\phi}^2\rangle + \mathcal{O}(|\Delta|^2, \bar{\Delta}^2) = 2\bar{\Delta}G(x_1, x_2) + \cdots \tag{4.12}$$

where $G(x, y) = \langle\phi(x)\bar{\phi}(y)\rangle$. We will see this more generally below, but this simple example already reminds us that holomorphicity of correlators of chiral operators fails when SUSY is spontaneously or explicitly broken.

Why, then, did the superfield arguments carry through? The point is that we were always asking explicitly holomorphic questions. Since we are treating the SUSY-breaking parameter as a component of a nonpropagating superfield, the superfield action is still broken up into a D-term part integrated over $d^4\theta$ and an F-term part integrated over $d^2\theta$. The latter will remain holomorphic in fields, in particular in the coupling superfields. The former can contain terms that mimic F-terms after expanding in $\Delta, \bar{\Delta}$ and integrating over $d^2\bar{\theta}$ [15]. However these terms are proportional to $\bar{\Delta}$ and do not contribute in the $\bar{\Delta} \to 0$ limit. So we may compute the F-term by setting the antiholomorphic soft breaking parameter $\bar{\Delta}$ to zero, and so isolating the purely holomorphic dependence of the chiral correlators. All of this will make sense in perturbation theory. We will have to be more careful when making nonperturbative arguments.

4.3 Anomaly arguments

In addition to the perturbative aruments in [77, 78], Cachazo et al. have given a non-perturbative argument for the Dijkgraaf-Vafa conjecture. Let us outline these steps, in order to highlight the places that they can fail when supersymmetry is broken.

The first important point is that correlators of gauge-invariant chiral operators are independent of position, and therefore (by cluster decomposition) factorize. The basis of this argument is the simple observation [79, 81, 78]:

$$\frac{\partial}{\partial x_1^\mu}\langle 0|\mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)|0\rangle = \sigma_{\mu,\alpha\dot{\alpha}}\langle 0|\left\{\bar{Q}^{\dot{\alpha}}, [Q^\alpha, \mathcal{O}(x_1)]\right\}\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)|0\rangle = 0\,. \tag{4.13}$$

This vanishes because the operators are chiral and so $\bar{Q}$ on both sides of the anticommutator can be pushed through the other operators and made to act on the vacuum. So long as the vacuum is supersymmetric, $\bar{Q}$ annihilates the vacuum and the derivative vanishes.

The second important claim is that the correlators of chiral operators are holomorphic in the couplings. More precisely, if λ is some superpotential coupling for the term $\int d^4x\, d^2\theta\, \lambda\mathcal{O}_\lambda$, then

$$\frac{\partial}{\partial\bar{\lambda}}G(x_1, \ldots x_n) = \langle 0|\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\int d^4x\left\{\bar{Q}, [\bar{Q}, \bar{\mathcal{O}}_{\bar{\lambda}}]\right\}|0\rangle \tag{4.14}$$

is only requred to vanish when $\bar{Q}$ (anti-) commutes with the chiral operators *and* annihilates the vacuum. The proof in [78] depends on the statement that for some coupling g_i in $W(\Phi)$, the derivatives of the low-energy superpotential can be written as:

$$\frac{\partial W_{\rm eff}(g_k, S, \ldots)}{\partial g_i} = \left\langle \frac{\partial W(\Phi)}{\partial g_i} \right\rangle . \tag{4.15}$$

This statement depends on the right hand side being a holomorphic function of g_i.

The last ingredient of this proof which requires a SUSY-invariant vacuum involves the Konishi anomaly, reflecting the anomalous variation of the measure of the path integral for Φ under the field redefinition

$$\Phi \longrightarrow \Phi + \delta\Phi = \Phi + f(\Phi, \mathcal{W}_\alpha) . \tag{4.16}$$

Explicitly, the current generating this transformation

$$J_f = {\rm Tr}\, \bar{\Phi} e^V f(\Phi, \mathcal{W}_\alpha) , \tag{4.17}$$

where V is the real superfield for the U(N) vector multiplet, satisfies an anomalous conservation law:

$$\bar{D}^2 J_f = {\rm Tr}\, f \frac{\partial W(\Phi)}{\partial \Phi} + \frac{1}{32\pi^2} \sum_{ij} \left[\mathcal{W}_\alpha, \left[\mathcal{W}^\alpha, \frac{\partial f}{\partial \Phi_{ij}} \right] \right]_{ji} \tag{4.18}$$

where the indices i, j are U(N) adjoint indices. Since $\bar{D}^2 J$ can be written as a $\bar{Q}$-commutator, upon taking the expectation value of (4.18), the left hand side vanishes, if the vacuum is supersymmetric. With a judicious choice of f, and using the factorization of chiral correlators, Cachazo et al. then map the resulting equation to the loop equations of the matrix model described above.

These three points in the argument of [78] fail when the vacuum is not supersymmetric. However, if we add the soft breaking term discussed in the previous section, the arguments hold if we restrict ourselves to the parts of the chiral correlators that depend solely on the holomorphic coupling Δ and not on the antiholomorphic coupling $\bar{\Delta}$. The point is that if we promote an F-term coupling to a superfield whose auxiliary component has vev Δ, the perturbation to the lagrangian taks the form:

$$\delta L = \int d^4x \left(\Delta p(\phi) + \bar{\Delta} \bar{p}(\bar{\phi}) \right) \tag{4.19}$$

where ϕ denotes the bottom components of any of the chiral superfields that appeared in this perturbation. Chiral correlators

$$G(x_1, \ldots, x_n) = \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle \tag{4.20}$$

are deformed to:

$$G_{\Delta,\bar{\Delta}} = \left\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) e^{-\left(\int \Delta p(\phi) - \int \bar{\Delta} \bar{p}(\bar{\phi})\right)} \right\rangle . \tag{4.21}$$

In general there will be a non-trivial dependence on both Δ and $\bar{\Delta}$. However, if we expand $\exp\left\{-\int\left(\Delta p(\phi)+\bar{\Delta}\bar{p}(\bar{\phi})\right)\right\}$ in $\bar{\Delta}$ and keep the $\bar{\Delta}$-independent term, the resulting correlator can be written as a correlator of chiral operators in the original vacuum,[17] for which the above anomaly-based arguments apply.

This argument works to all orders in a perturbation expansion in the soft coupling. One might worry that nonperturbative terms in $\bar{\Delta}$ would make it difficult to separate out the holomorphic part of a correlator. However, in general one does not expect nonperturbative behavior in the coefficients of relevant operators (which is precisely what characterizes these soft terms), as long as they do not change the large-field behavior of the action. Non-analytic behavior in a coupling occurs when the vev of the operator it multiplies is comparable to the vevs of other terms in the action. If there is a ϕ^4 term in the potential energy in addition to the soft breaking term, then this marginal term dominates over the soft-breaking term in the action. Said another way, one may try to apply Dyson's argument [92] for the breakdown of perturbation theory at large orders. Applied to the coupling which dominates at large field values — e.g. the quartic scalar coupling in a renormalizable field theory — it states that since the theory is unstable for negative values of the coupling, it must have a singularity when the coupling vanishes. Therefore the perturbation series diverges. But if one flips the sign of a m^2 term in the presence of a quartic coupling, physics is not singular and so perturbation theory in a soft mass $(\delta m)^2$ should converge (there is potentially an infrared divergence, but we are assuming the unperturbed supersymmetric mass of Φ is nonzero).[18]

One *will* have to worry if the explicit SUSY breaking terms include marginal or large irrelevant couplings that change the asymptotics of field space. At this point the correlators can be nonperturbative in the SUSY-breaking parameters and one cannot meaningfully extract the holomorphic piece. This should not come as a surprise, as the ultraviolet theory will depend strongly on the SUSY-breaking parameters in such a case.

5. Conclusion

In this paper, we have made a stringy identification of the auxiliary fields of $\mathcal{N}=2$ multiplets in type II on a CY. This identification extends our knowledge of bosonic couplings between sectors of string theory to $\mathcal{N}=2$ multiplets. Specifically, closed-string multiplets act as spurion superfields for open-string modes, and, in a local model, spurions for localized closed-string modes are closed-string fields which are not normalizable. This latter fact fits nicely with the modularity of physics made possible by these constructions. The

[17] The vacuum will of course be modified by the addition of the soft-breaking terms. However, the following argument shows that this does not affect the $\bar{\Delta}$-independent part of chiral correlators. The perturbed vacuum can be calculated perturbatively in $\Delta,\bar{\Delta}$ about the SUSY vacuum. Since, as we will argue below, this perturbation series is expected to be analytic when the SUSY breaking is soft, we can isolate the terms which are independent of $\bar{\Delta}$. These terms will be states which are connected to the unperturbed vacuum by chiral operators, and the anomaly argument goes through.

[18] We thank S. Shenker for a discussion on this point, and for reminding us of Dyson's argument.

couplings of the localized modes performing some service to physics are determined by closed-string modes which also have support far away in the CY; it is the vevs of auxiliary fields for these modes that transmit SUSY-breaking between modules.

Our discussion has covered what should be a 'fundamental domain' for the action of mirror symmetry (to the extent that mirror symmetry is a relevant concept in the presence of magnetic fluxes). The set of objects which can arise as auxiliary fields in this context is the same as the set of objects which can appear as central charges of the supersymmetry algebra. Heuristically, this is because they both arise by acting with two supercharges on the lowest component of a supermultiplet.

A fact which is highlighted by our work is that in type-II string theory, we do not understand precisely what all of these objects are. In particular, we have been able to understand auxiliary fields in the vector multiplets in type-IIB string theory, and auxiliary fields in the hypermultiplets in type-IIA string theory. Both involve NS-NS three-form flux. In addition to RR and NSNS fluxes, NUT charges [93] can also play the role of auxiliary fields for vectors in type IIA and hypers in type IIB [43]. The value of such a charge can be thought of, at least heuristically, as a number of KK monopoles, which are related by U-duality to D-branes and NS-branes.

Still, the *generic* object which can play the role of a central charge in type-II string theory is not understood. This becomes clear in a semi-flat approximation to the geometry as a flat T^3 fibration [94]. This approxiation is good near a large complex structure point. In this description, the central charges are encoded in the monodromies acting on the string theory on the T^3 fibers, associated to homotopy generators of the base. For example, the fact that N units of flux F are supported near a singular fiber is encoded by the fact that the corresponding potential C ($F = dC$) undergoes a shift through N periods $C \to C + 2\pi N$ during a tour around that point in the base.

In this approximation, mirror symmetry is visible as an element of the duality group of the T^3 fibers. Conjugating the monodromy group of a generic IIB solution with only RR and NSNS fluxes by this element leads to a set of central charges which are not merely curvatures and fluxes. The monodromy group will generically include non-geometric elements of the U-duality group (such as T-dualities), and even non-perturbative ones (such as S-dualities). Such string backgrounds, studied in e.g. [95], can often be considered resolutions of asymmetric orbifolds. In this sense, the mirror of even the deformed conifold, with NSNS 3-form flux through the 3-sphere, is non-geometric [96].

The identification of the relevant central charges would be useful at least because it would help in identifying the domain walls which change their values. For the case of RR and NSNS flux vacua, D5- and NS5-branes on holomorphic curves provide BPS domain walls between vacua with different values of RR and NSNS fluxes [28], and non-BPS domain walls between vacua with different amounts of supersymmetry [97]. As an example of the utility of these domain walls, the fact that they interpolate between SUSY vacua with different fluxes provides an alternative derivation of the GVW superpotential [28]. It would be quite useful to understand microscopically the domain walls between vacua with different values of the other kinds of central charges.

We hope to shed some light on these questions in upcoming work [45].

Acknowledgments

We would like to thank A. Adams, N. Arkani-Hamed, M. Berkooz, N. Berkovits, O. DeWolfe, S. Hellerman, S. Kachru, S. Katz, M. Peskin, L. Randall, H. Schnitzer, M. Schulz, S. Shenker, R. Tatar, T. Taylor and N. Wyllard for helpful discussions and comments. The research of A. Lawrence is supported in part by NSF grant PHY-0331516. J. McGreevy is supported by a Princeton University Dicke Fellowship, and by the Department of Energy under Grant No. DE-FG03-92ER40701.

A.L. would like to thank the SLAC Theory Group and the Stanford ITP for their hospitality during an early stage of this project. He would also like to thank the Institute for Studies in Theoretical Physics and Mathematics (IPM) in Tehran, and especially Shirin Davarpanah and Maryam Soltani, for running the ISS2003 school in Anzali, Iran so smoothly that it was actually possible for the organizers to get work done. Finally, he would like to thank the Aspen Center for Physics and the Kavli Institute for Theoretical Physics at UCSB, where much of this work was carried out.

References

[1] J. Wess and J. Bagger, *Supersymmetry and supergravity*, 2nd ed., Princeton University Press, Princeton 1992.

[2] M. Dine and N. Seiberg, *Couplings and scales in superstring models*, *Phys. Rev. Lett.* **55** (1985) 366.

[3] V.S. Kaplunovsky, *Mass scales of the string unification*, *Phys. Rev. Lett.* **55** (1985) 1036.

[4] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

[5] T. Banks and M. Dine, *Couplings and scales in strongly coupled heterotic string theory*, *Nucl. Phys.* **B 479** (1996) 173 [hep-th/9605136].

[6] N. Kaloper, M. Kleban, A.E. Lawrence and S. Shenker, *Signatures of short distance physics in the cosmic microwave background*, *Phys. Rev.* **D 66** (2002) 123510 [hep-th/0201158].

[7] S. Dimopoulos and H. Georgi, *Softly broken supersymmetry and* SU(5), *Nucl. Phys.* **B 193** (1981) 150.

[8] L. Girardello and M.T. Grisaru, *Soft breaking of supersymmetry*, *Nucl. Phys.* **B 194** (1982) 65.

[9] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240];
S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S.P. Trivedi, *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055].

[10] M. Dine and D. MacIntire, *Supersymmetry, naturalness and dynamical supersymmetry breaking*, *Phys. Rev.* **D 46** (1992) 2594 [hep-ph/9205227];
L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B 557** (1999) 79 [hep-th/9810155];
G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *J. High Energy Phys.* **12** (1998) 027 [hep-ph/9810442].

JHEP06(2004)007

[11] Z. Chacko, M.A. Luty and E. Ponton, *Massive higher-dimensional gauge fields as messengers of supersymmetry breaking*, *J. High Energy Phys.* **07** (2000) 036 [hep-ph/9909248];
D.E. Kaplan, G.D. Kribs and M. Schmaltz, *Supersymmetry breaking through transparent extra dimensions*, *Phys. Rev.* **D 62** (2000) 035010 [hep-ph/9911293];
Z. Chacko, M.A. Luty, A.E. Nelson and E. Ponton, *Gaugino mediated supersymmetry breaking*, *J. High Energy Phys.* **01** (2000) 003 [hep-ph/9911323];
M. Schmaltz and W. Skiba, *Minimal gaugino mediation*, *Phys. Rev.* **D 62** (2000) 095005 [hep-ph/0001172];
D.E. Kaplan and G.D. Kribs, *Gaugino-assisted anomaly mediation*, *J. High Energy Phys.* **09** (2000) 048 [hep-ph/0009195].

[12] S. Dimopoulos, S. Kachru, N. Kaloper, A.E. Lawrence and E. Silverstein, *Small numbers from tunneling between brane throats*, *Phys. Rev.* **D 64** (2001) 121702 [hep-th/0104239]; *Generating small numbers by tunneling in multi-throat compactifications*, hep-th/0106128.

[13] N. Berkovits and W. Siegel, *Superspace effective actions for 4d compactifications of heterotic and type-II superstrings*, *Nucl. Phys.* **B 462** (1996) 213 [hep-th/9510106].

[14] N. Berkovits, *Covariant quantization of the Green-Schwarz superstring in a Calabi-Yau background*, *Nucl. Phys.* **B 431** (1994) 258 [hep-th/9404162].

[15] N.J. Evans, S.D.H. Hsu and M. Schwetz, *Exact results in softly broken supersymmetric models*, *Phys. Lett.* **B 355** (1995) 475 [hep-th/9503186];
N.J. Evans, S.D.H. Hsu, M. Schwetz and S.B. Selipsky, *Exact results and soft breaking masses in supersymmetric gauge theory*, *Nucl. Phys.* **B 456** (1995) 205 [hep-th/9508002];
N.J. Evans, S.D.H. Hsu and M. Schwetz, *Controlled soft breaking of $N = 1$ SQCD*, *Phys. Lett.* **B 404** (1997) 77 [hep-th/9703197].

[16] O. Aharony, J. Sonnenschein, M.E. Peskin and S. Yankielowicz, *Exotic nonsupersymmetric gauge dynamics from supersymmetric QCD*, *Phys. Rev.* **D 52** (1995) 6157 [hep-th/9507013];
E. D'Hoker, Y. Mimura and N. Sakai, *Gauge symmetry breaking through soft masses in supersymmetric gauge theories*, *Phys. Rev.* **D 54** (1996) 7724 [hep-th/9603206];
H.-C. Cheng and Y. Shadmi, *Duality in the presence of supersymmetry breaking*, *Nucl. Phys.* **B 531** (1998) 125 [hep-th/9801146];
N. Arkani-Hamed and R. Rattazzi, *Exact results for non-holomorphic masses in softly broken supersymmetric gauge theories*, *Phys. Lett.* **B 454** (1999) 290 [hep-th/9804068];
M.A. Luty and R. Rattazzi, *Soft supersymmetry breaking in deformed moduli spaces, conformal theories and $N = 2$ Yang-Mills theory*, *J. High Energy Phys.* **11** (1999) 001 [hep-th/9908085];
G.R. Farrar, G. Gabadadze and M. Schwetz, *The spectrum of softly broken $N = 1$ supersymmetric Yang-Mills theory*, *Phys. Rev.* **D 60** (1999) 035002 [hep-th/9806204].

[17] R. Dijkgraaf and C. Vafa, *Matrix models, topological strings and supersymmetric gauge theories*, *Nucl. Phys.* **B 644** (2002) 3 [hep-th/0206255]; *On geometry and matrix models*, *Nucl. Phys.* **B 644** (2002) 21 [hep-th/0207106]; *A perturbative window into non-perturbative physics*, hep-th/0208048.

[18] P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, *Nucl. Phys.* **B 593** (2001) 99 [hep-th/0003198].

[19] A. Anisimov, M. Dine, M. Graesser and S. Thomas, *Brane world SUSY breaking*, *Phys. Rev.* **D 65** (2002) 105011 [hep-th/0111235]; *Brane world SUSY breaking from string/M theory*, *J. High Energy Phys.* **03** (2002) 036 [hep-th/0201256].

JHEP06(2004)007

[20] M. Graña, *D3-brane action in a supergravity background: the fermionic story*, *Phys. Rev.* **D 66** (2002) 045014 [hep-th/0202118].

[21] O. DeWolfe and S.B. Giddings, *Scales and hierarchies in warped compactifications and brane worlds*, *Phys. Rev.* **D 67** (2003) 066008 [hep-th/0208123].

[22] M. Graña, *MSSM parameters from supergravity backgrounds*, *Phys. Rev.* **D 67** (2003) 066006 [hep-th/0209200].

[23] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241].

[24] M. Graña, T.W. Grimm, H. Jockers and J. Louis, *Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes*, *Nucl. Phys.* **B 690** (2004) 21 [hep-th/0312232].

[25] N.J. Evans, M. Petrini and A. Zaffaroni, *The gravity dual of softly broken $N = 1$ super Yang-Mills*, *J. High Energy Phys.* **06** (2002) 004 [hep-th/0203203];
O. Aharony, E. Schreiber and J. Sonnenschein, *Stable non-supersymmetric supergravity solutions from deformations of the maldacena-Núñez background*, *J. High Energy Phys.* **04** (2002) 011 [hep-th/0201224];
V. Borokhov and S.S. Gubser, *Non-supersymmetric deformations of the dual of a confining gauge theory*, *J. High Energy Phys.* **05** (2003) 034 [hep-th/0206098];
S. Kuperstein and J. Sonnenschein, *Analytic non-supersymmtric background dual of a confining gauge theory and the corresponding plane wave theory of hadrons*, *J. High Energy Phys.* **02** (2004) 015 [hep-th/0309011].

[26] L. Alvarez-Gaumé and M. Mariño, *Softly broken $N = 2$ QCD*, hep-th/9606168; *More on softly broken $N = 2$ QCD*, *Int. J. Mod. Phys.* **A 12** (1997) 975 [hep-th/9606191].

[27] C. Vafa, *Superstrings and topological strings at large-N*, *J. Math. Phys.* **42** (2001) 2798 [hep-th/0008142].

[28] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070], erratum *ibid.* **B 608** (2001) 477.,

[29] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[30] N. Berkovits, H. Ooguri and C. Vafa, *On the worldsheet derivation of large-N dualities for the superstring*, hep-th/0310118.

[31] R. Gopakumar and C. Vafa, *Topological gravity as large-N topological gauge theory*, *Adv. Theor. Math. Phys.* **2** (1998) 413 [hep-th/9802016]; *On the gauge theory/geometry correspondence*, *Adv. Theor. Math. Phys.* **3** (1999) 1415 [hep-th/9811131];
J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure $N = 1$ super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001];
I.R. Klebanov and A.A. Tseytlin, *Gravity duals of supersymmetric $\mathrm{SU}(N) \times \mathrm{SU}(N + M)$ gauge theories*, *Nucl. Phys.* **B 578** (2000) 123 [hep-th/0002159];
J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[32] F. Cachazo, K.A. Intriligator and C. Vafa, *A large-N duality via a geometric transition*, *Nucl. Phys.* **B 603** (2001) 3 [hep-th/0103067].

[33] I. Brunner, M.R. Douglas, A.E. Lawrence and C. Romelsberger, *D-branes on the quintic*, *J. High Energy Phys.* **08** (2000) 015 [hep-th/9906200].

[34] M.R. Douglas, *Topics in D-geometry*, *Class. and Quant. Grav.* **17** (2000) 1057 [hep-th/9910170];
M.R. Douglas, B. Fiol and C. Romelsberger, *The spectrum of BPS branes on a noncompact Calabi-Yau*, hep-th/0003263;
D.-E. Diaconescu and M.R. Douglas, *D-branes on stringy Calabi-Yau manifolds*, hep-th/0006224;
M.R. Douglas, *D-branes on Calabi-Yau manifolds*, math.AG/0009209; *D-branes and $N = 1$ supersymmetry*, hep-th/0105014.

[35] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, *Kodaira-Spencer theory of gravity and exact results for quantum string amplitudes*, *Commun. Math. Phys.* **165** (1994) 311 [hep-th/9309140].

[36] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, *Holomorphic anomalies in topological field theories*, *Nucl. Phys.* **B 405** (1993) 279 [hep-th/9302103].

[37] M. Klein, *Couplings in pseudo-supersymmetry*, *Phys. Rev.* **D 66** (2002) 055009 [hep-th/0205300];
C.P. Burgess, E. Filotas, M. Klein and F. Quevedo, *Low-energy brane-world effective actions and partial supersymmetry breaking*, *J. High Energy Phys.* **10** (2003) 041 [hep-th/0209190].

[38] R. Grimm, M. Sohnius and J. Wess, *Extended supersymmetry and gauge theories*, *Nucl. Phys.* **B 133** (1978) 275.

[39] B. de Wit and J.W. van Holten, *Multiplets of linearized* SO(2) *supergravity*, *Nucl. Phys.* **B 155** (1979) 530.

[40] M. de Roo, J.W. van Holten, B. de Wit and A. Van Proeyen, *Chiral superfields in $N = 2$ supergravity*, *Nucl. Phys.* **B 173** (1980) 175.

[41] J.J. Atick, L.J. Dixon and A. Sen, *String calculation of Fayet-Iliopoulos D terms in arbitrary supersymmetric compactifications*, *Nucl. Phys.* **B 292** (1987) 109.

[42] M. Dine, I. Ichinose and N. Seiberg, *F terms and D terms in string theory*, *Nucl. Phys.* **B 293** (1987) 253.

[43] S. Kachru, M.B. Schulz, P.K. Tripathy and S.P. Trivedi, *New supersymmetric string compactifications*, *J. High Energy Phys.* **03** (2003) 061 [hep-th/0211182].

[44] Discussions of mirror symmetry in the presence of NS flux can be found for example in
S. Gurrieri, J. Louis, A. Micu and D. Waldram, *Mirror symmetry in generalized Calabi-Yau compactifications*, *Nucl. Phys.* **B 654** (2003) 61 [hep-th/0211102];
S. Gurrieri and A. Micu, *Type iib theory on half-flat manifolds*, *Class. and Quant. Grav.* **20** (2003) 2181 [hep-th/0212278];
S. Fidanza, R. Minasian and A. Tomasiello, *Mirror symmetric* SU(3)*-structure manifolds with ns fluxes*, hep-th/0311122.

[45] S. Hellerman, A. Lawrence and J. McGreevy, work in progress.

[46] H. Partouche and B. Pioline, *Partial spontaneous breaking of global supersymmetry*, *Nucl. Phys.* **56B** *(Proc. Suppl.)* (1997) 322 [hep-th/9702115].

[47] P. Candelas, P.S. Green and T. Hubsch, *Rolling among Calabi-Yau vacua*, *Nucl. Phys.* **B 330** (1990) 49.

[48] P. Candelas and X. de la Ossa, *Moduli space of Calabi-Yau manifolds*, *Nucl. Phys.* **B 355** (1991) 455.

JHEP06(2004)007

[49] A. Strominger, *Special geometry*, *Commun. Math. Phys.* **133** (1990) 163.

[50] J. Polchinski, *String theory*, Cambridge University Press, Cambridge 1998.

[51] B.A. Ovrut, *Superstrings and the auxiliary field structure of supergravity*, *Phys. Lett.* **B 205** (1988) 455.

[52] S. Cecotti, S. Ferrara and L. Girardello, *Heterotic superstrings in auxiliary field background and extended space-time supersymmetry*, *Phys. Lett.* **B 206** (1988) 451.

[53] T. Banks, L.J. Dixon, D. Friedan and E.J. Martinec, *Phenomenology and conformal field theory or can string theory predict the weak mixing angle?*, *Nucl. Phys.* **B 299** (1988) 613.

[54] T. Banks and L.J. Dixon, *Constraints on string vacua with space-time supersymmetry*, *Nucl. Phys.* **B 307** (1988) 93.

[55] C.M. Hull, *Sigma model beta functions and string compactifications*, *Nucl. Phys.* **B 267** (1986) 266.

[56] J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure $N = 1$ super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001].

[57] J.H. Schwarz, *Covariant field equations of chiral $N = 2$ $D = 10$ supergravity*, *Nucl. Phys.* **B 226** (1983) 269.

[58] S.F. Hassan, *T-duality, space-time spinors and RR fields in curved backgrounds*, *Nucl. Phys.* **B 568** (2000) 145 [hep-th/9907152].

[59] M. Grana and J. Polchinski, *Supersymmetric three-form flux perturbations on AdS_5*, *Phys. Rev.* **D 63** (2001) 026001 [hep-th/0009211].

[60] S.S. Gubser, *Supersymmetry and F-theory realization of the deformed conifold with three-form flux*, hep-th/0010010.

[61] M. Grana and J. Polchinski, *Gauge/gravity duals with holomorphic dilaton*, *Phys. Rev.* **D 65** (2002) 126005 [hep-th/0106014].

[62] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[63] I. Antoniadis, H. Partouche and T.R. Taylor, *Spontaneous breaking of $N = 2$ global supersymmetry*, *Phys. Lett.* **B 372** (1996) 83 [hep-th/9512006].

[64] I. Antoniadis and T.R. Taylor, *Dual $N = 2$ SUSY breaking*, *Fortschr. Phys.* **44** (1996) 487 [hep-th/9604062].

[65] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227].

[66] E. Witten, *Baryons and branes in anti de Sitter space*, *J. High Energy Phys.* **07** (1998) 006 [hep-th/9805112];
D.J. Gross and H. Ooguri, *Aspects of large-N gauge theory dynamics as seen by string theory*, *Phys. Rev.* **D 58** (1998) 106002 [hep-th/9805129].

[67] N. Seiberg and E. Witten, *Electric-magnetic duality, monopole condensation and confinement in $N = 2$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 426** (1994) 19 [hep-th/9407087], erratum *ibid.* **B 430** (1994) 485.

[68] B.R. Greene, D.R. Morrison and C. Vafa, *A geometric realization of confinement*, *Nucl. Phys.* **B 481** (1996) 513 [hep-th/9608039].

[69] S. Kachru, S. Katz, A.E. Lawrence and J. McGreevy, *Open string instantons and superpotentials*, *Phys. Rev.* **D 62** (2000) 026001 [hep-th/9912151].

[70] M.R. Douglas, *D-branes, categories and $N = 1$ supersymmetry*, *J. Math. Phys.* **42** (2001) 2818 [hep-th/0011017].

[71] E. Witten, *Branes and the dynamics of QCD*, *Nucl. Phys.* **B 507** (1997) 658 [hep-th/9706109].

[72] M. Reid, *Minimal models of canonical 3-folds*, Advanced Studies in Pure Mathematics 1, S. Iitaka ed., Kinokuniya 1983, pp. 131–180.

[73] F. Cachazo and C. Vafa, *$N = 1$ and $N = 2$ geometry from fluxes*, hep-th/0206017.

[74] P. Candelas, X. De La Ossa, A. Font, S. Katz and D.R. Morrison, *Mirror symmetry for two parameter models, I*, *Nucl. Phys.* **B 416** (1994) 481 [hep-th/9308083].

[75] S. Katz, D.R. Morrison and M. Ronen Plesser, *Enhanced gauge symmetry in type-II string theory*, *Nucl. Phys.* **B 477** (1996) 105 [hep-th/9601108].

[76] S. Kachru, S. Katz, A.E. Lawrence and J. McGreevy, *Mirror symmetry for open strings*, *Phys. Rev.* **D 62** (2000) 126005 [hep-th/0006047]; *Mirror symmetry for open strings*, *Phys. Rev.* **D 62** (2000) 126005 [hep-th/0006047].

[77] R. Dijkgraaf, M.T. Grisaru, C.S. Lam, C. Vafa and D. Zanon, *Perturbative computation of glueball superpotentials*, *Phys. Lett.* **B 573** (2003) 138 [hep-th/0211017].

[78] F. Cachazo, M.R. Douglas, N. Seiberg and E. Witten, *Chiral rings and anomalies in supersymmetric gauge theory*, *J. High Energy Phys.* **12** (2002) 071 [hep-th/0211170].

[79] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Instanton effects in supersymmetric theories*, *Nucl. Phys.* **B 229** (1983) 407.

[80] G.C. Rossi and G. Veneziano, *Nonperturbative breakdown of the nonrenormalization theorem in supersymmetric QCD*, *Phys. Lett.* **B 138** (1984) 195.

[81] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, *Nonperturbative aspects in supersymmetric gauge theories*, *Phys. Rept.* **162** (1988) 169.

[82] A. Brandhuber, H. Ita, H. Nieder, Y. Oz and C. Romelsberger, *Chiral rings, superpotentials and the vacuum structure of $N = 1$ supersymmetric gauge theories*, *Adv. Theor. Math. Phys.* **7** (2003) 269 [hep-th/0303001].

[83] K.-I. Izawa and T. Yanagida, *Dynamical supersymmetry breaking in vector-like gauge theories*, *Prog. Theor. Phys.* **95** (1996) 829 [hep-th/9602180];
K.A. Intriligator and S. Thomas, *Dynamical supersymmetry breaking on quantum moduli spaces*, *Nucl. Phys.* **B 473** (1996) 121 [hep-th/9603158]; *Dual descriptions of supersymmetry breaking*, hep-th/9608046.

[84] K. Konishi and G. Veneziano, *Effective action for dynamical supersymmetry breaking*, *Phys. Lett.* **B 187** (1987) 106.

[85] G. Veneziano and S. Yankielowicz, *An effective lagrangian for the pure $N = 1$ supersymmetric Yang-Mills theory*, *Phys. Lett.* **B 113** (1982) 231.

[86] H. Ooguri and C. Vafa, *Worldsheet derivation of a large-N duality*, *Nucl. Phys.* **B 641** (2002) 3 [hep-th/0205297].

[87] R. Dijkgraaf, S. Gukov, V.A. Kazakov and C. Vafa, *Perturbative analysis of gauged matrix models*, *Phys. Rev.* **D 68** (2003) 045007 [hep-th/0210238].

[88] E. Witten, *Constraints on supersymmetry breaking*, *Nucl. Phys.* **B 202** (1982) 253.

[89] T.R. Taylor, G. Veneziano and S. Yankielowicz, *Supersymmetric QCD and its massless limit: an effective lagrangian analysis*, *Nucl. Phys.* **B 218** (1983) 493;
M. Peskin, in *Problems in unification and supergravity*, G. Farrar and F. Heney eds., AIP, New York 1984.

[90] V. Balasubramanian, J. de Boer, B. Feng, Y.-H. He, M.-x. Huang, V. Jejjala, and A. Naqvi, *Multi-trace superpotentials vs. matrix models*, *Commun. Math. Phys.* **242** (2003) 361 [hep-th/0212082].

[91] M.T. Grisaru and D. Zanon, *Covariant supergraphs, 1. Yang-Mills theory*, *Nucl. Phys.* **B 252** (1985) 578.

[92] F. Dyson, *Divergence of perturbation theory in quantum electrodynamics*, *Phys. Rev.* **85** (1952) 631.

[93] C.M. Hull, *Gravitational duality, branes and charges*, *Nucl. Phys.* **B 509** (1998) 216 [hep-th/9705162].

[94] A. Strominger, S.-T. Yau and E. Zaslow, *Mirror symmetry is T-duality*, *Nucl. Phys.* **B 479** (1996) 243 [hep-th/9606040].

[95] S. Hellerman, J. McGreevy and B. Williams, *Geometric constructions of nongeometric string theories*, *J. High Energy Phys.* **01** (2004) 024 [hep-th/0208174];
A. Dabholkar and C. Hull, *Duality twists, orbifolds and fluxes*, *J. High Energy Phys.* **09** (2003) 054 [hep-th/0210209];
A. Flournoy, B. Wecht and B. Williams, to appear.

[96] S. Hellerman, Princeton journal club.

[97] S. Kachru, X. Liu, M.B. Schulz and S.P. Trivedi, *Supersymmetry changing bubbles in string theory*, *J. High Energy Phys.* **05** (2003) 014 [hep-th/0205108].